THE ASTRONOMICAL
JOURNAL

PUBLISHED BY THE AMERICAN INSTITUTE OF PHYSICS
FOR THE AMERICAN ASTRONOMICAL SOCIETY

VOLUME 65 1960 April ~ No. 1278 *NUMBER 3*

Positions, Intensities, and Sizes of Bright Celestial Sources at a Wavelength of 10.2 Cm

R. M. Sloanaker and J. H. Nichols
Radio Astronomy Branch, U. S. Naval Research Laboratory, Washington, D. C.

(Received January 15, 1960)

The positions of eight bright discrete sources at 10.2 cm have been measured to estimated accuracies of ±1′ pe using the Naval Research Laboratory 84-ft antenna. The equivalent Gaussian diameters and flux densities were also determined.

INTRODUCTION

EXPERIENCE during 1958 with the new NRL 84-ft antenna at 10 cm suggested the possibility of determining the positions of the brighter discrete sources to an accuracy of the order of ±1′. Therefore, during January, 1959, the positions of eight of the brightest discrete sources at 10.2-cm wavelength were measured. The eight selected sources (see Table I) are scattered fairly uniformly over the range of declination from +59° to −42°, and thus cover in declination most of the sky visible from the latitude of Washington, D. C.

The antenna temperatures and apparent angular diameters of the sources were measured concurrently with the position measurements. The results are given in the present paper.

EQUIPMENT

Radiometer

The radiometer was designed around the Dicke (1946) system. No input preselection was used, and the radiometer accepted power in two frequency bands, each 5.5 Mc wide and separated by 60 Mc about the center frequency, which was set at 2930 Mc (10.2 cm). The output time constant of the radiometer was set at 2.2 sec. The double sideband receiver noise factor measured at the antenna input terminal was 4.4 in units of power ratio. The measured peak-to-peak output fluctuation level was 1.5°C and the measured rms output fluctuation was 0.5°C, when the receiver input was terminated in a matched load at 290°K.

The calibration standards for the measurement of received power were two thermal noise sources consisting of matched resistive terminations in waveguide immersed in two agitated water baths at different temperatures. These thermal noise sources were substituted in turn for the feed horn and used to calibrate noise power from an argon gas discharge tube that was coupled into the antenna waveguide feed line through a directional coupler. The coupled noise power from the argon tube corresponded to a measured equivalent temperature of 122.0°C, and was used to calibrate the radiometer at frequent intervals during measurements.

Antenna

The measurements were made with the NRL 84-ft paraboloidal antenna located at Maryland Point, Maryland, at longitude 77°13′57″.4, and latitude 38°22′26″.1. A plane-polarized horn feed antenna, connected to the radiometer input by about 2 ft of waveguide, was located at the focus of the reflector and oriented with the electric vector in the declination direction. The illumination of the edge of the paraboloid was about 14 db below that at the vertex. At 10.2 cm the aperture efficiency of the antenna with this feed horn is approximately 38.5%, based on the spectrum of Cygnus-A obtained from measurements with the NRL 50-ft antenna at wavelengths of 3.15 cm, Haddock and McCullough (1954), Mayer, McCullough, and Sloanaker (1957); 9.4 cm, Haddock, Mayer, and Sloanaker (1954); and 21 cm, Hagen, McClain, and Hepburn (1954). Antenna patterns were measured

TABLE I. Basic position data—dial positions corrected only for standard optical refraction.

Source	Date 1959	R. A. (Dial position) h	m	s	n	σ_1 s	Dec. (Dial position) °	′	n	σ_1 ′	Atm pressure mb	Air temp. °F	Rel humidity %	Wind mph
Cassiopeia-A	Jan. 20	23	21	34.4	8	2.5	58	33.6	5	0.5	1020	45	84	2 N
Cygnus-A	Jan. 20	19	58	04.0	3	0	40	35.8	3	0.2	1022	40	98	2 W
Taurus-A	Jan. 7	05	32	08.0	3	0.5	21	57.3	3	0.1	1021	35	45	0
Virgo-A	Jan. 8	12	28	47.1	3	0.6	12	35.0	4	0.2	1020	27	63	0
Orion nebula	Jan. 7	05	33	17.4	4	1.5	−05	26.9	3	0.2	1021	35	45	0
Omega nebula	Jan. 15	18	18	05.8	4	4.0	−16	13.9	4	1.5	1006	48	93	7 S
Omega nebula	Jan. 16	18	18	05.8	4	2.4	−16	13.0	4	0.7	993	35	92	2 NW
Omega nebula	Average	18	18	05.8	8	2.6	−16	13.4	8	1.1				
Sagittarius-A	Jan. 15	17	43	01.9	4	1.6	−28	58.8	4	0.2	1006	48	93	7 S
Sagittarius-A	Jan. 16	17	43	01.0	3	3.5	−28	58.4	4	0.3	993	35	92	5 NW
Sagittarius-A	Average	17	43	01.5	7	2.4	−28	58.6	8	0.4				
Centaurus-A	Jan. 8	13	22	56.9	5	0.5	−42	48.6	5	0.8	1021	26	63	0

prior to the position measurements by making scans in right ascension and declination through the three brightest discrete sources, Cassiopeia-A, Cygnus-A, and Taurus-A. The scans gave identical patterns to within a few tenths minutes of arc in half-width and shape. The measured half-intensity widths of these patterns were 18ʹ.2 in right ascension and 18ʹ.6 in declination.

The half-intensity widths of these sources as measured at long wavelengths are roughly 4ʹ, 2ʹ, and 5ʹ, respectively (Pawsey 1955). The half-intensity width D of a pattern obtained by scanning a source having a Gaussian brightness distribution of half-width S with a Gaussian antenna beam of half-width B is given by the equation

$$D^2 = B^2 + S^2. \tag{1}$$

Therefore, a source with Gaussian brightness distribution of half-intensity width equal to 4ʹ would broaden by 0ʹ.4 a scan for a Gaussian antenna beam having a half-power width of 18ʹ.4; accordingly the scan patterns obtained from these sources may be broader than the antenna pattern by a few tenths minutes of arc. Figure 1 shows the scan patterns obtained for the Cassiopeia-A source in right ascension and declination. The dashed curves in the figure are Gaussian curves of the same half-widths. Comparisons of the Cassiopeia-A scans with the symmetrical Gaussian curves show that the patterns are symmetrical out to a diameter of 22ʹ and are slightly asymmetrical beyond this diameter.

The antenna itself, described in detail elsewhere (McClain 1958), has the following features pertinent to the present measurements. The 84-ft diam aluminum reflector is equatorially mounted. A sidereal drive is incorporated in the polar axis system. In addition, in both polar and declination axes a differential scan mode permits any angular velocity from about ±0.1°/min to ±1°/min to be imparted.

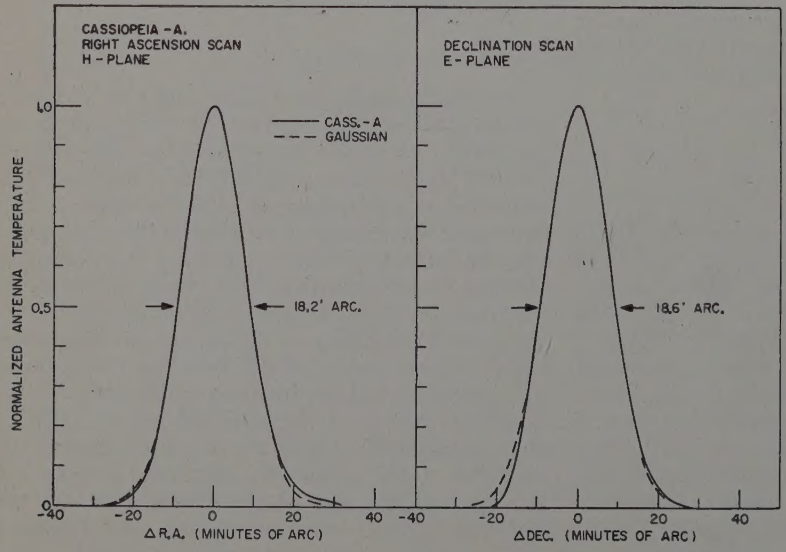


FIG. 1. Antenna patterns in right ascension and declination from Cassiopeia–A scans.

POSITION MEASUREMENTS

All position measurements were made in the same manner. The peak of the antenna beam was slowly scanned through the source alternately in right ascension and declination. The center position of each scan curve was determined while it was being made. After each scan, the antenna was brought to the center position before the next scan in the other coordinate was made. This process was repeated until it was obvious that a position very close (within less than $1'$) to the center position had been found. Then the peak of the beam, from about 0.6 peak power level up, was again scanned north through the source in declination at a rate of about $1'/4$ sec. The center declination of this scan curve was read at several power levels and averaged to obtain the measured declination of the source. The antenna was then brought to this declination and driven ahead of the source in hour angle and stopped. The antenna beam then drifted through the source at the sidereal rate, again from about the 0.6 peak power level up. The right ascension of the source was calculated from the time of the center of the drift scan determined at several power levels. The positions thus obtained were considered to constitute one independent measurement of dial declination and dial right ascension.

All of the measurements were made either at night or on overcast cloudy days to eliminate the effect of differential heating of the antenna tower by the sun. As nearly as possible, the measurements were made at times of minimum wind velocity. All measurements were made within $0^{h}30^{m}$ of the meridian to reduce pointing errors that might be caused by improper orientation of the polar axis. All except a few of the measurements were made within 20^{m} of the meridian. No systematic trend of apparent position was detected over this small range of hour angle and each source was measured both East and West of the meridian.

In Table I listing the results of the basic position measurements and other pertinent data appear the average dial right ascensions and declinations that were obtained for each day's measurements and the grand average when measurements were made on more than one day. The positions have been corrected only for optical refraction for standard atmospheric conditions of 760-mm-Hg total pressure and 10°C temperature. n is the number of independent position measurements and σ_1 is the standard deviation of a single measured dial position from the corresponding average dial position. Also listed are the total atmospheric pressure in millibars, the air temperature in degrees Fahrenheit, and the percent of relative humidity at the times of the measurements.

CORRECTION FOR RADIOMETER TIME CONSTANT

The response of the radiometer was identical in form to the response of a single RC circuit with a time constant of 2.2 sec. It can be shown that the effect of a radiometer time constant that is very small compared to an input Gaussian drift scan function is merely to delay the output scan curve by a time equal to the time constant, with essentially no distortion in the shape of the curve, and that this delay is independent of the scan rate as long as the time constant is short compared to the scan time.

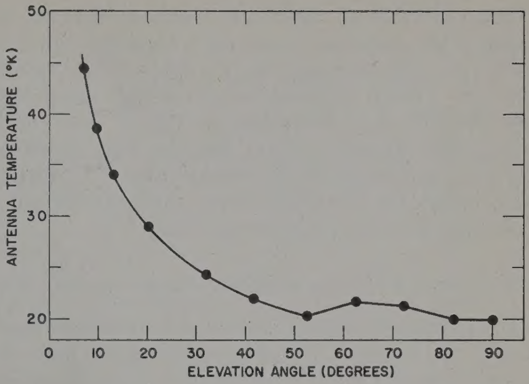


FIG. 2. Apparent sky antenna temperature (°K) vs elevation angle (deg).

Therefore, the measured dial right ascensions given in Table I must be corrected by -2.2 sec to correct for the time constant. In the measurement of declination the antenna beam was scanned North through the sources at the rate of $1'/4$ sec; therefore the measured dial declinations must be corrected by $0\overset{\prime}{.}5$ South.

CORRECTION FOR DIFFERENTIAL ATMOSPHERIC EMISSION AND GROUND RADIATION OVER A SOURCE

Figure 2 shows the apparent sky antenna temperature that was measured as a function of elevation angle when the antenna was scanned along the meridian. Part of the apparent sky radiation is radiation by the atmosphere, and part of it is ground radiation picked up in the antenna sidelobes. Measurements of position in which the antenna is moved in elevation angle will be shifted from the true positions by the background variation. Based on the measured source antenna temperatures and half-widths (Table IV) and the measured sky antenna temperature variation, the error in the measured declination is negligible for all of the sources except Centaurus-A. For Centaurus-A the error is $-0\overset{\prime}{.}2$, and therefore the measured dial declination must be corrected by $+0\overset{\prime}{.}2$.

DIFFERENTIAL ATMOSPHERIC ABSORPTION OVER A SOURCE

Ionospheric absorption should be negligible at 10 cm (Millman 1958); however, tropospheric absorption by oxygen, water vapor, and suspended particles is appreciable at small elevation angles. For an extended source, differential absorption across the source increases the elevation angle of the apparent center of the source. Calculations based on the measured source diameters (Table IV) and on atmospheric absorption calculated for an absorption of 1% at the zenith (Muller and Westerhout 1957, Shulkin 1951) show that this effect is negligible for all of the sources. The greatest effect occurs for Centaurus-A for which the difference in absorption across the source is 0.05%.

CORRECTION FOR ATMOSPHERIC RADIO REFRACTION

The measured dial declinations and, to a very small extent, the measured dial right ascensions, must be corrected for the effect of atmospheric radio refraction which differs from optical refraction because of the effect of water vapor. These corrections were made as follows:

The radio index of refraction of the air near the ground was calculated for each measurement period from the weather data listed in Table I using the formula developed by Smith and Weintraub (1953):

$$(n_0-1)\cdot 10^6=\frac{77.6}{T}\left(p+4810\frac{e}{T}\right), \tag{2}$$

where n_0 is the index of refraction of the air near the ground, p is the total atmospheric pressure in millibars, T is the absolute temperature of the air, and e is the partial pressure of the water vapor present in the air in millibars. e is a function of the relative humidity given by

$$e=RH\times p_S, \tag{3}$$

where RH is the relative humidity expressed as a decimal, and p_S is the pressure of saturated water vapor at the same temperature, T. According to Smith and Weintraub, Eq. (2) is accurate to within 0.5% in (n_0-1) for frequencies up to 30 000 Mc for all normally encountered values of T, p, and e.

To calculate the total refractive bending of a radio wave passing completely through the earth's atmosphere Shulkin (1952) has derived the formula

$$R=(n_0-1)\cot\theta_0-\int_{(\cot\theta)_{n=1}}^{\cot\theta_0}(n-1)d(\cot\theta), \tag{4}$$

where R is the refractive bending in elevation angle; n_0 is, as before, the radio index of refraction of the air at ground level; θ_0 is the apparent elevation of the source for an observer on the ground; θ is the apparent elevation of the source for an observer at a height H above the earth's surface; n is the index of refraction at height H above the earth's surface; and $(\cot\theta)_{n=1}$ is the value of $\cot\theta$ at the height above the earth's surface where n is equal to 1.0. Shulkin has developed a method by which the integral in Eq. (4) can be evaluated from radio-sonde data taking into account the effect of the earth's curvature. Based on typical radio-sonde data, Shulkin shows that the error caused by neglecting the integral term in Eq. (4) is less than 3.5% in R for a source at an apparent elevation angle of 10°, and that this error decreases rapidly with increasing elevation angle.

Therefore, since all of the sources, except Centaurus-A, were measured at elevation angles much greater than 10°, the radio refractive bending in elevation angle was calculated for these sources using the formula

$$R=(n_0-1)\cot\theta_0. \tag{5}$$

However, for the measurement of Centaurus-A, which was made at an elevation angle of 8.8°, the effect of earth curvature cannot be neglected, and Eq. (5) is not valid.

Based on Shulkin's formulas, Bean and Cahoon (1957) have calculated the total radio refractive bending for low elevation angles for a wide variety of radiosonde data, and from these calculations have developed empirical formulas that may be used to calculate the refractive bending for low elevation angles. Using

TABLE II. Corrected position data, epoch 1959.0. Dial positions corrected for time constant, sky antenna temperature variation, radio refraction, aberration, nutation, and precession to 1959.0.

Source	Date 1959	Corrected mean, dial right ascension 1959.0			Corrected mean, dial declination 1959.0		Standard optical refraction	Radio refraction
		h	m	s	°	′	′	′
Cassiopeia-A	Jan. 20	23	21	32.3	58	32.8	0.36	0.41
Cygnus-A	Jan. 20	19	58	02.9	40	35.2	0.04	0
Taurus-A	Jan. 7	05	32	03.9	21	56.9	0.29	0.32
Virgo-A	Jan. 8	12	28	44.4	12	34.6	0.46	0.52
Orion nebula	Jan. 7	05	33	13.5	−05	27.3	0.94	1.02
Omega nebula	Jan. 15	18	18	04.6	−16	14.7	1.38	1.61
Omega nebula	Jan. 16	18	18	04.6	−16	13.7	1.38	1.51
Omega nebula	Average	18	18	04.6	−16	14.2	(1.38)	(1.56)
Sagittarius-A	Jan. 15	17	43	00.5	−28	59.9	2.30	2.71
Sagittarius-A	Jan. 16	17	42	59.6	−28	59.3	2.30	2.55
Sagittarius-A	Average	17	43	00.1	−28	59.6	(2.30)	(2.63)
Centaurus-A	Jan. 8	13	22	53.8	−42	49.9	5.92	6.72

TABLE III. Optical positions—epoch 1959·0.

Source	R.A. 1959.0 h m s	Dec. 1959·0 ° ′	Optical position minus dial position, Table II Δ(R.A.) ′	 Δ(R.A.) s	 Δ(Dec.) ′
Cassiopeia-A[a]	23 21 33.8	58 35.42	0.2	1.5	2.6
Cygnus-A[b]	19 58 03.2	40 37.25	0.1	0.3	2.0
Taurus-A[b]	05 32 02.5	21 59.7	−0.3	−1.5	2.8
Virgo-A[b]	12 28 45	12 37.1	0.1	0.5	2.5
Theta–One Orionis[c]	05 33 15.5	−05 24.78	0.5	2	2.5
Centaurus-A[b]	13 23 00	−42 48.4	1.1	6	1.5

[a] Minkowski (1959). [b] Baade and Minkowski (1954). [c] Strand (1958).

these formulas, the total radio refraction for our measurements of Centaurus-A is 6′.81.

The values of radio refraction calculated for our measurements are listed in Table II. For comparison, Table II also lists optical refraction for standard conditions (760 mm Hg and 10°C) obtained from Allen (1955).

For small elevation angles, where earth curvature is important, the total radio refraction R may also be calculated by a formula due to Pearcey (1947).

For Centaurus-A on the meridian, Pearcey's formula gave R equal to 6′.72. As mentioned above, the method due to Bean and Cahoon gave R equal to 6′.81 for Centaurus-A and hence the two methods agree to within 0′.1.

For our measurements, the greatest difference between optical and radio refraction was 0′.80 (for Centaurus-A) and calculations showed that to correct the average dial declination listed in Table I further for radio refraction, it was sufficiently accurate (to within less than 0′.1 in declination) merely to correct the average dial declinations of Table I by the difference between standard optical refraction and radio refraction for the source on the meridian. Also it was found that to within an accuracy of $0^{\mathrm{s}}.1$ of time no further correction of the average dial right ascensions of Table I was necessary for radio refraction.

For a source having a finite extent in elevation, differential refraction across the source increases the elevation of the apparent center of area of the source. The difference in refraction across the half-intensity width of Centaurus-A is about 0′.05, and is much less for the other sources.

IONOSPHERIC REFRACTION

Bailey (1948) has derived an expression for the refraction of radio waves passing through an ionized layer that has a parabolic distribution of electron density with height.

To obtain an estimate of the magnitude of ionospheric refraction at 2930 Mc, we assume a horizontally stratified ionosphere consisting of a single parabolic layer of maximum electron density equal to 10^6 electrons per cm^3 at a height of 300 km and of total thickness equal to 500 km. From Bailey's equation the total refraction for this ionospheric layer at 2930 Mc for an elevation angle of 9° (Centaurus-A, roughly) is 0′.02. Therefore, we regard ionospheric refraction as negligible at 2930 Mc.

CORRECTION FOR ABERRATION, NUTATION, AND PRECESSION

After the average dial positions had been corrected as discussed above they were further corrected for aberration, nutation, and precession to 1959.0 using the standard astronomical formulas. The largest correction for the combined effect of aberration, nutation, and precession amounted to $1^{\mathrm{s}}.89$ in right ascension (for Taurus-A) and to 0′.3 in declination (for Cassiopeia-A).

CORRECTED DIAL POSITIONS

Table II lists the measured average dial right ascensions and dial declinations corrected for receiver time constant, variation of sky background antenna temperature with elevation, radio refraction, aberration, nutation, and precession to 1959.0.

It should be emphasized that the average corrected dial positions listed in Table II are *not* absolute positions. These dial positions are expected to contain systematic errors caused by such things as incorrect orientation of the polar axis of the antenna, nonperpendicularity of the declination and polar axes of the antenna, incorrect setting of the zeros of the right ascension and declination dials, etc. The determination of the systematic errors is discussed below in connection with the optical positions of the sources.

RANDOM POSITION ERRORS

Based on the standard deviations for single measurements (σ_1, Table I), the probable error in the average dial positions is ±0′.2 in right ascension and ±0′.3 in declination.

Based on the work of Bean and Cahoon (1957), Shulkin (1952), and Smith and Weintraub (1953), the estimated probable error in the calculated radio refraction is ±4% and is due mainly to uncertainties in the meteorological factors. This corresponds to $\pm 0\overset{\prime}{.}3$ in refraction for Centaurus-A and negligible errors for the other sources.

Except for the refraction corrections, the corrections applied to the data are small, and it is assumed that the errors in these corrections have a negligible effect on the present results.

COMPARISON WITH OPTICAL POSITIONS

Table III lists the mean optical positions for Epoch 1959.0 for the six sources for which accurate optical positions are available. The optical identifications for these sources, with the exception of the Orion Nebula source, are given by Baade and Minkowski (1954). For the Orion Nebula source, Table III lists the mean optical position of θ-1 Orionis, the exciting stars, obtained from Strand (1958). The optical identification for the Orion Nebula source is given by Haddock, Mayer, and Sloanaker (1954).

We calculate the arithmetic differences between the optical positions of Table III and the corresponding dial positions of Table II. We define:

$$\Delta(\text{R.A.}) = (\text{Optical right ascension, Table III}) \quad \textit{Minus} \quad (\text{average corrected dial right ascension for the same source, Table II}). \tag{6}$$

$$\Delta(\text{Dec.}) = (\text{Optical declination, Table III}) \quad \textit{Minus} \quad (\text{average corrected dial declination for the same source, Table II}). \tag{7}$$

The values of Δ(R.A.) and Δ(Dec.) are listed in Table III, and plotted vs declination in Figs. 3(a) and 3(b). Table III also lists Δ(R.A.) in minutes of arc subtended at the antenna. The solid curve in Fig. 3(a) is a least-squares fit of Bessel's formula to the measured values (see the following). The mean values of Δ(R.A.) and Δ(Dec.) are $1\overset{s}{.}5$ and $2\overset{\prime}{.}3$, respectively, and the standard deviations of the individual values are $2\overset{s}{.}5$ and $0\overset{\prime}{.}5$. Inspection of Fig. 3 shows that the dial positions are systematically shifted with respect to the optical positions. These shifts can be accounted for by errors in the antenna dial zero settings (to be described), since any differences between the absolute radio and optical positions should tend to be random.

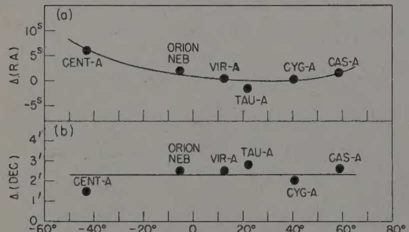


FIG. 3. Optical positions *minus* corrected dial positions. (a) Difference in right ascension vs declination; (b) difference in declination vs declination.

The order of magnitude of the probable agreement between the absolute radio and optical positions can be inferred from the values of Δ(R.A.) and Δ(Dec.). The maximum spread in the values is $1\overset{\prime}{.}4$ (or $7\overset{s}{.}5$) in Δ(R.A) and $1\overset{\prime}{.}3$ in Δ(Dec.), in both cases between Centaurus-A and Taurus-A. In the absence of compensating errors, systematic variations with declination in antenna errors would increase the total spreads. As discussed below, systematic antenna errors may account for most of the spread in the Δ(R.A.) values. The random errors in the measured dial positions discussed previously would also tend to increase the spread.

Therefore, it is conservative to conclude that the absolute radio positions agree with the optical positions for all of the sources to within probable errors given by ±[one-half of maximum spread in Δ(R.A.) and Δ(Dec.) *plus* the random errors in the dial positions, added directly]. Since the errors determined in this manner are not calculated errors in the statistical sense, we will call them estimated probable errors.

On this basis, it is concluded from the data that the absolute 2930 Mc positions agree with the optical positions of the sources listed in Table III to within $\pm 1'$ estimated pe in both right ascension and declination except for Centaurus-A, for which the estimated probable error in declination is $\pm 1\overset{\prime}{.}3$ because of the additional error in refraction. The estimated probable errors are in all cases larger than the sum of the random dial errors and the deviations of the individual Δ(R.A.) and Δ(Dec.) values from their respective mean values.

SYSTEMATIC ANTENNA ERRORS

Optical tests indicated that the antenna polar axis alignment was better than $0\overset{\prime}{.}5$, and that the polar and declination axes were perpendicular to within $1'$ (McClain 1958). Previous to the radio measurements, the hour angle and declination dials were set to read the direction of the geometrical axis of the antenna aperture using optical surveying methods. The error in the dial settings is estimated to be $\pm 2'$ pe. Optical measurements indicated that the feed horn was centered on the antenna aperture to within less than $\frac{1}{16}$ in. An error of $\frac{1}{16}$ in. would produce a pointing error of about $2'$. This information is all that is definitely known about the possible systematic antenna errors except for what may be deduced from the radio measurements.

The effect of the most common types of antenna errors on the measured right ascensions can be expressed by Bessel's formula (Smart 1956):

$$t = a + b\tan(\text{Dec.}) + c\sec(\text{Dec.}) \tag{8}$$

where t is the error in right ascension as a function of declination produced by antenna errors represented by the constants a, b, and c. a is the error in the setting of

TABLE IV. Absolute positions at 2930 mc, epoch 1959.0, antenna temperatures, equivalent gaussian source half-widths, and flux densities.

Source	R.A. (1959.0)	Dec. (1959.0)	Antenna temp. °C	Gaussian half-widths R.A.	Gaussian half-widths Dec.	Flux densities $\times 10^{25}$ w m^{-2} (cps)$^{-1}$
Cassiopeia-A	optical±7.5$^{\rm s}$ pe[a]	optical±1′ pe[a]	89±2 pe	point[c]	point[c]	124±11 pe
Cygnus-A	optical±5$^{\rm s}$ [a]	optical±1′ [a]	45±1	point[c]	point[c]	63±6
Taurus-A	optical±4.5$^{\rm s}$ [a]	optical±1′ [a]	48±1	point[c]	point[c]	67±6
Virgo-A	optical±4$^{\rm s}$ [a]	optical±1′ [a]	7·5±0.4	point[c]	point[c]	10±1
Orion nebula	optical±4$^{\rm s}$ [a]	optical±1′ [a]	25±0.5	7′ ±$0'.5$ pe	7′ ±1′ pe	40±4
Omega nebula	$18^{\rm h}18^{\rm m}6.1^{\rm s}\pm4^{\rm s}$	$-16^\circ11'.9\pm1'$	35±0.7	$7'.5\pm0'.5$	$8'.5\pm1'$	58±5
Sagittarius-A[b]	$17^{\rm h}43^{\rm m}1.6^{\rm s}\pm4{\cdot}5^{\rm s}$	$-28^\circ57'.3\pm1'$	28±0.6	14′ ±$0'.3$	16′ ±$0'.5$	65±7
Centaurus-A	optical±5.5$^{\rm s}$ [a]	optical±$1'.3$[a]	10±0.4	8′ ±$0'.5$	5′ ±$1'.5$	16±2

[a] Consistent with optical position (listed in Table III) to within est pe indicated. [b] Bright central part only. [c] Unresolved point source.

the hour angle dial zero plus components due to the errors in the orientation of the polar axis and the nonperpendicularity of the axes; b is made up of components due to the error in the orientation of the polar axis and the nonperpendicularity of the axes; and c is the difference in the hour angle coordinate between the direction of the antenna beam and the geometrical axis of the aperture.

A least-squares fit of Bessel's formula has been calculated for the values of Δ(R.A.) vs declination (Table III). The least-squares coefficients are $a=-5^{\rm s}.9$, $b=-3^{\rm s}.3$, and $c=6^{\rm s}.6$. Errors of these magnitudes in the antenna would produce negligible systematic variation in declination. The least-squares values of Δ(R.A.) are plotted vs declination in Fig. 3(a) (solid curve). The maximum deviation of the measured Δ(R.A.) values from the least-square curve is $1^{\rm s}.5$ (for Taurus-A). If we based our estimate of the systematic antenna errors on the least-squares Bessel's formula, we could conclude that the absolute radio and optical right ascensions agree to within about $0'.5$, and since these antenna errors would produce negligible errors in declination, we could also reduce the estimated probable error in the agreement in declination. However, the least-squares coefficients of Bessel's formula are largely determined by the values of Δ(R.A.) for Cassiopeia-A and Centaurus-A [see Fig. 3(a)]; therefore, the use of Bessel's formula to estimate systematic errors will strongly depend on the assumption that the radio and optical positions of these two sources are identical. For this reason, we will not make use of Bessel's formula to reduce the errors estimated in the preceding section.

It should be noted that Centaurus-A is complex at long radio wavelengths. Shain (1958) has found that at 19.7 Mc Centaurus-A consists of a bright central source of small extent surrounded by an extended source with dimensions of about $7^\circ.5$ by $2^\circ.5$. Spectra given by Shain show that the contribution from the extended source should be small at 2930 Mc for an 18′ antenna pattern. The present measurements do not show an extended Centaurus-A source, and the measured half-intensity diameter of about 7′ (Table IV) is in good agreement with the optical size of the bright central part of the nebula.

POSITIONS OF SAGITTARIUS–A AND OMEGA NEBULA

The absolute 2930 Mc positions of Sagittarius-A and the Omega Nebula listed in Table IV were obtained by correcting the dial declinations of Table II by $+2'.3$, the average value of Δ(Dec.) obtained from comparison of the dial and optical positions; and by correcting the dial right ascensions by $+1^{\rm s}.5$, the average value of Δ(R.A.). The right ascensions determined in this manner differ by less than $0'.3$ from those that would be obtained using Bessel's formula. The uncertainties in the positions are $\pm1'$ est pe in both right ascension and declination.

INTENSITY AND SIZE MEASUREMENTS

A large number of complete right ascension drift scans and declination scans were made through the peak position of each source in addition to the position measurements. The average antenna temperatures (Table IV) of the sources were determined from the right ascension drift scan amplitudes calibrated with the noise power from the argon gas discharge tube, and were corrected for receiver nonlinearity and also for atmospheric absorption calculated for an absorption of 1% at the zenith. The apparent angular diameters of the sources (Table IV) were calculated from the half-intensity widths of the scans using Eq. (1), on the assumption that the brightness distributions across the sources are Gaussian. Flux densities (Table IV) were calculated from the corrected antenna temperatures using the equation

$$S=\left(\frac{2kT_a}{KA}\right)\left(1+\frac{d^2}{B^2}\right). \tag{9}$$

In Eq. (9), k is Boltzmann's constant, T_a is the antenna temperature, K is the antenna aperture efficiency ($K=0.385$), A is the area of the antenna aperture, d is the half-intensity width of the source brightness distribution, and B is the antenna beam half-width.

The second factor in Eq. (9), Seeger, Westerhout, and Van de Hulst (1956), is the correction factor for finite source size and is based on the assumption that the antenna beam and source brightness distributions are Gaussian.

RESULTS

The 2930 Mc positions and estimated errors are given in Table IV. For those sources for which the optical positions are the best estimates for the radio positions, the probable errors on the agreement of the radio and optical positions are given in Table IV.

Table IV also lists the equivalent Gaussian half-intensity widths, antenna temperatures, and flux densities for the sources. The antenna temperatures have been corrected for receiver nonlinearity and atmospheric absorption. The flux densities were calculated for an estimated antenna aperture efficiency of 0.385±0.031 pe and have been corrected for source size on the assumption that the source brightness distributions are Gaussian.

REFERENCES

Allen, C. W. 1955, *Astrophysical Quantities* (The Athlone Press, London), p. 115.

Baade, W. and Minkowski, R. 1954, *Astrophys. J.* **119**, 206, 215.

Bailey, D. K. 1948, *Terrestrial Magnetism and Atmospheric Elec.* **53**, 41.

Bean, B. R. and Cahoon, B. A. 1957, *Proc. Inst. Radio. Engrs.* **45**, 1545.

Dicke, R. H. 1946 *Rev. Sci. Instr.* **17**, 268.

Haddock, F. T., Mayer, C. H., and Sloanaker, R. M. 1954, *Nature* **174**, 176.

Haddock, F. T. and McCullough, T. P. 1954, private communication.

Hagen, J. P., McClain, E. F., and Hepburn, N. 1954, *Proc. Inst. Radio Engrs.* **42**, 1811.

Mayer, C. H., McCullough, T. P., and Sloanaker, R. M. 1957, unpublished.

McClain, E. F. 1958, *Sky and Telescope* **17**, No. 12, 608.

Millman, G. H. 1958, *Proc. Inst. Radio Engrs.* **46**, 1492.

Minkowski, R. 1959, *Paris Symposium on Radio Astronomy* (IAU Symposium No. 9 and URSI Symposium No. 1, 1958, Stanford University Press), p. 315.

Muller, C. A. and Westerhout, G. 1957, *Bull. Astron. Insts. Netherlands* **13**, 151.

Pawsey, J. L. 1955, *Astrophys. J.* **121**, 1.

Pearcey, T. 1947, cited from: McCready, L. L., Pawsey, J. L., and Payne-Scott, R. 1947, *Proc. Roy. Soc.* (London) **A190**, 357.

Seeger, Ch. L., Westerhout, G., and Van de Hulst, H. C. 1956, *Bull. Astron. Insts. Netherlands* **13**, 89.

Shain, C. A. 1958, *Australian J. Phys.* **2**, 517.

Shulkin, M. 1951, *U. S. Nav. Res. Lab. Rept.* 3843.

——. 1952, *Proc. Inst. Radio Engrs.* **40**, 554.

Smart, W. M. 1956, *Textbook on Spherical Astronomy* (Cambridge University Press) 4th ed., p. 80.

Smith, E. K. and Weintraub, S. 1953, *Proc. Inst. Radio Engrs.* **41**, 1035.

Strand, K. Aa. 1958, *Astrophys. J.* **128**, 14.

THE ASTRONOMICAL JOURNAL VOLUME 65, NUMBER 3 APRIL, 1960

New Set of Variables for Astronomical Problems

A. M. Garofalo
Missile & Space Vehicle Department, General Electric Company, Philadelphia, Pennsylvania

(Received September 21, 1959)

The equations of motion for a particle A under the gravitational influence of n other particles in a system of reference with origin at one of these particles B are written in terms of new variables. These have the classical property of being constant if we consider only the influence of B on the motion of A. In addition, they have the advantage of not introducing any new singularities if the instantaneous conic of A relative to B does not degenerate into a straight line.

INTRODUCTION

THE object of this paper is to write Newton's equations of motion for a particle, under the gravitational influence of n other particles, which we shall hereafter call bodies, and other external forces in terms of new variables which will "subtract out" the unperturbed conic section of the particle orbit relative to one of the bodies and which have other properties not shared by such generalized variables as the Keplerian elements. A brief outline showing how the standard equations in terms of the usual generalized variables can be derived will establish the relationship between this work and what has been done in the past.

In a coordinate system with the origin at one of the bodies of mass M, which will hereafter be called the M body, and axes parallel to an inertial set of axes, the equations of motion for the particle can be written as follows:

$$\frac{d^2r_i}{dt^2}+\kappa(M+m)\frac{r_i}{|\mathbf{r}|^3}=\frac{\partial R}{\partial r_i}+\frac{F_i}{m}, \tag{1}$$

where

- κ is the gravitational constant,
- R is the disturbing function associated with the $n-1$ other bodies,
- m is the mass of the particle,
- F_i are the components of the nongravitational forces,
- r_i are the components of the position vector $\mathbf{r}$ of the particle in the coordinate system described above, and
- $|\mathbf{r}|$ is the scalar $(r_1^2+r_2^2+r_3^2)^{\frac{1}{2}}$.

Let α_k represent a set of six generalized variables, and let

$$\begin{aligned} r_i&=f_i(\alpha_k,t), \quad i=1,2,3; \quad k=1,2,\cdots 6\\ \dot{r}_i&=g_i(\alpha_k,t), \end{aligned} \tag{2}$$

where the dot represents a derivative with respect to time. If we imagine that the solution to Eq. (1) is known and that the sytem (2) can be inverted to give α_k as functions of r_i, $\dot{r}_i$, and t, we can write the α_k as explicit functions of the time,

$$\alpha_k=\alpha_k[r_i(t),\dot{r}_i(t),t]=\alpha_k(t). \tag{3}$$

Since we are actually interested in the total dependence of the α_k on the time and not in the α_k as functions of r_i, $\dot{r}_i$, and t obtained by inverting Eq. (2), we do not have to concern ourselves with the inversion of the system (2) but shall only require that the functions f_i and g_i have continuous first partial derivatives in a certain domain. We can then write the differential equations for the α_k that these functions imply, and from the differential equations establish the properties of the α_k.

The functions f_i and g_i are selected, in practice, so that if the α_k are all constants, f_i and g_i are the solution and the derivative of the solution of

$$\frac{d^2r_i}{dt^2}+\kappa(M+m)\frac{r_i}{|\mathbf{r}|^3}=0; \tag{4}$$

that is, the equation where the masses of all bodies other than the M body are set equal to zero and there are no nongravitational forces. This implies that

$$\begin{aligned} \frac{dr_i}{dt}&=\left(\frac{\partial f_i}{\partial t}\right)_{\alpha_k},\\ \left(\frac{\partial \dot{r}_i}{\partial t}\right)_{\alpha_k}&=\left(\frac{\partial g_i}{\partial t}\right)_{\alpha_k}=-\kappa(M+m)\frac{r_i}{|\mathbf{r}|^3}, \end{aligned} \tag{5}$$

or alternatively

$$\frac{\partial f_i}{\partial \alpha_k}\dot{\alpha}_k=0,$$

$$\frac{\partial g_i}{\partial \alpha_k}\dot{\alpha}_k=\frac{\partial R}{\partial r_i}+\frac{F_i}{m},$$

where we have used the summation convention.

Techniques have been developed in celestial mechanics to invert the matrix, multiplying the $\dot{\alpha}_k$. Since for the two-body problem the time enters into the functions f_i and g_i in a rather complicated way, it is usual to introduce the eccentric anonomaly E in case the unperturbed orbit is an ellipse, and to relate the time and E by means of Kepler's equation.

If Ω, i, ω, a, e, and χ are selected as the generalized variables, we notice that three singularities occur in the differential equations for this sytem. These singularities, occurring at $i=0$, $e=0$, and $e=1$, would be removable only if ∇R and F_i vanished at these values. The first two can in fact be traced physically (see Smart's *Celestial Mechanics* for the definition of these quantities and the differential equations) to the indeterminateness of the line of nodes when $i=0$ and the indeterminateness of the angle between the line of nodes and the line of apsides when $e=0$. The third singularity apparently arises through the way time was introduced; the unperturbed orbit is considered to be an ellipse and time is introduced through the parametric equations for an ellipse which are no longer valid when the orbit becomes a parabola. Therefore, it would seem advisable (i) to introduce time in a different fashion, and (ii) not to use the angles Ω, i, and ω. Also, in order to obtain some type of symmetry it would seem advisable to use the same type of variables, e.g., we might expect to obtain a more symmetrical set of equations if we used a and b (the major and minor semi-axes) instead of a and e.

NEW VARIABLES

Let us divide our six generalized variables into two sets of three each, which we shall call α_l and γ_l. Let us define the α_l, the γ_l, and another variable θ implicitly through the sytem

$$\begin{aligned} r_i&=\mathfrak{L}_{ij}(\alpha_l)\rho_j(\theta,\gamma_l), \quad i,j,l=1,2,3\\ \dot{r}_i&=\mathfrak{L}_{ij}(\alpha_l)\eta_j(\theta,\gamma_l), \qquad (6)\\ \dot{\theta}&=f(\theta,\gamma_l), \quad \theta=0 \quad \text{at} \quad t=t_1, \end{aligned}$$

where $\mathfrak{L}_{ij}$ is an orthogonal matrix such that $\boldsymbol{\rho}$ and $\boldsymbol{\eta}$ are in the new xy plane, $\boldsymbol{\rho}+\boldsymbol{\eta}$ is directed along the new positive z axis, and at $t=t_1$, $\boldsymbol{\rho}$ is directed along the new positive x axis. Its elements are dependent on three independent angles, such as the Eulerian angles that define the instantaneous plane and the orientation of the new x axis. However, the meaning of the α_l need not be specified at this stage. If as before we imagine that we can set $\gamma_l=\gamma_l[\theta,r_i(t),\dot{r}_i(t)]$ and substitute this in the differential equation for $\dot{\theta}$, we can integrate this equation to obtain $\theta=\theta(t)$.

As is well known, a conic section in polar coordinates can be represented by an equation of the form

$$1/\rho=(\mu/h^2)+\beta_1\cos\theta+\beta_2\sin\theta, \qquad (7)$$

where β_1, β_2, and h shall stand for our set γ_l and $\mu=\kappa(M+m)$. The eccentricity e for such an ellipse is given by

$$e=(h^2/\mu)(\beta_1^2+\beta_2^2)^{\frac{1}{2}}$$

and ϕ, the angle between the positive x axis; the perihelion is defined by

$$\sin\phi=\frac{\beta_2}{(\beta_1^2+\beta_2^2)^{\frac{1}{2}}} \quad \text{and} \quad \cos\phi=\frac{\beta_1}{(\beta_1^2+\beta_2^2)^{\frac{1}{2}}}.$$

Therefore we could write Eq. (7) in the equivalent form

$$1/\rho=(\mu/h^2)[1+e\cos(\theta-\phi)],$$

We shall let

$$\begin{aligned} \rho_1&=\rho\cos\theta,\\ \rho_2&=\rho\sin\theta, \qquad (8\\ \rho_3&=0, \end{aligned}$$

where, of course, ρ is considered as a function o h, β_1, β_2, and θ as defined by Eq. (7).

In addition we shall set

$$\dot{\theta}=h/\rho^2. \qquad (9$$

Differentiating Eq. (8) partially with respect to θ, we obtain

$$\begin{aligned} \frac{\partial\rho_1}{\partial\theta}&=-\rho^2\left(\beta_2+\frac{\mu}{h^2}\sin\theta\right),\\ \frac{\partial\rho_2}{\partial\theta}&=\rho^2\left(\beta_1+\frac{\mu}{h^2}\cos\theta\right), \qquad (10\\ \frac{\partial\rho_3}{\partial\theta}&=0. \end{aligned}$$

Let $\eta_j=(\partial\rho_j/\partial\theta)\dot{\theta}$, where η_j is expressed as a functio of h, β_1, β_2, and θ; i.e., if we consider all the α_l and γ constant, then $\mathfrak{L}_{ij}\eta_j=(\partial/\partial t)(\mathfrak{L}_{ij}\rho_j)$.
Thus

$$\begin{aligned} \eta_1&=-[\beta_2+(\mu/h^2)\sin\theta]h,\\ \eta_2&=[\beta_1+(\mu/h^2)\cos\theta]h, \qquad (11\\ \eta_3&=0. \end{aligned}$$

We shall also want to make use of the fact that ther exists a matrix $\mathfrak{Q}$ such that $\dot{\mathfrak{L}}_{ij}=\mathfrak{L}_{ik}\mathfrak{Q}_{kj}$, where $\mathfrak{Q}$ $=-\mathfrak{Q}_{ts}$. This follows simply from differentiation of th equation $\tilde{\mathfrak{L}}\mathfrak{L}=\mathfrak{U}$ where $\mathfrak{U}$ is the unit matrix and th tilde ($\sim$) over a matrix represents the transpose of th matrix, or in our case the inverse of $\mathfrak{L}$:

$$\frac{d}{dt}(\tilde{\mathfrak{L}}\mathfrak{L})=0=\tilde{\mathfrak{L}}\dot{\mathfrak{L}}+\left(\frac{d}{dt}\tilde{\mathfrak{L}}\right)\mathfrak{L}$$

or

$$\tilde{\mathfrak{L}}\dot{\mathfrak{L}}=-\left(\frac{d}{dt}\tilde{\mathfrak{L}}\right)\mathfrak{L}=-\text{the transpose of }(\tilde{\mathfrak{L}}\dot{\mathfrak{L}})=\mathfrak{Q}. \qquad (12$$

Therefore, $\dot{\mathfrak{L}}=\mathfrak{L}\mathfrak{Q}$. We shall not specify the element α_l but shall use expression (12) when differentiatin the matrix $\mathfrak{L}$.

Let us now derive equations for $\dot{\gamma}_l$ and the thre independent components of $\mathfrak{Q}$ in terms of α_l, γ_l, θ, an the perturbations. Differentiating the first set of equations in Eq. (6) with respect to time, we obtain,

$$\frac{dr_i}{dt}=\mathfrak{L}_{ij}\mathfrak{Q}_{jk}\rho_k+\mathfrak{L}_{ij}\frac{\partial\rho_j}{\partial\theta}\dot{\theta}+\mathfrak{L}_{ij}\frac{\partial\rho_j}{\partial\gamma_l}\dot{\gamma}_l;$$

using the second set, we obtain

$$\frac{d\mathbf{r}_i}{dt}=\mathcal{L}_{ij}\eta_j=\mathcal{L}_{ij}\frac{\partial\rho_j}{\partial\theta}\dot{\theta}.$$

Thus,

$$\mathcal{L}_{ij}\mathcal{Q}_{jk}\rho_k+\mathcal{L}_{ij}\frac{\partial\rho_j}{\partial\gamma_l}\dot{\gamma}_l=0. \tag{13}$$

Differentiation of the second set in (6) gives

$$\frac{d\dot{\mathbf{r}}_i}{dt}=\frac{d^2\mathbf{r}_i}{dt^2}=\mathcal{L}_{ij}\mathcal{Q}_{jk}\eta_k+\mathcal{L}_{ij}\frac{\partial\eta_j}{\partial\theta}\dot{\theta}+\mathcal{L}_{ij}\frac{\partial\eta_j}{\partial\gamma_l}\dot{\gamma}_l.$$

Since

$$\frac{\partial R}{\partial\mathbf{r}_i}=\mathcal{L}_{ij}\frac{\partial R}{\partial\rho_j},$$

$$F_i=\mathcal{L}_{ij}\tilde{\mathcal{L}}_{jk}F_k,$$

$$\frac{\mathbf{r}_i}{|\mathbf{r}|^3}=\frac{\mathcal{L}_{ij}\rho_j}{|\boldsymbol{\varrho}|^3},$$

Eq. (1) may be written,

$$\mathcal{L}_{ij}\mathcal{Q}_{jk}\eta_k+\mathcal{L}_{ij}\frac{\partial\eta_j}{\partial\gamma_l}\dot{\gamma}_l+\mathcal{L}_{ij}\frac{\partial\eta_j}{\partial\theta}\dot{\theta}$$
$$=\mathcal{L}_{ij}\frac{\partial R}{\partial\rho_j}+\mathcal{L}_{ij}\tilde{\mathcal{L}}_{jk}\frac{F_k}{m}-\frac{\mu\mathcal{L}_{ij}\rho_j}{|\boldsymbol{\varrho}|^3}.$$

But by straightforward differentiation of Eq. (11) and substitution of h/ρ^2 for $\dot{\theta}$, one obtains

$$\frac{\partial\eta_j}{\partial\theta}\dot{\theta}=-\frac{\mu\rho_j}{|\boldsymbol{\varrho}|^3},$$

or

$$\mathcal{L}_{ij}\mathcal{Q}_{jk}\eta_k+\mathcal{L}_{ij}\frac{\partial\eta_j}{\partial\gamma_l}\dot{\gamma}_l=\mathcal{L}_{ij}\frac{\partial R}{\partial\rho_j}+\mathcal{L}_{ij}\tilde{\mathcal{L}}_{jk}\frac{F_k}{m}. \tag{14}$$

If we let

$$S_j=\frac{\partial R}{\partial\rho_j}+\tilde{\mathcal{L}}_{jk}\frac{F_k}{m},$$

and multiply Eqs. (13) and (14) by $\tilde{\mathcal{L}}$ we obtain

$$\begin{aligned}\mathcal{Q}_{jk}\rho_k+\frac{\partial\rho_j}{\partial\gamma_l}\dot{\gamma}_l&=0,\\ \mathcal{Q}_{jk}\eta_k+\frac{\partial\eta_j}{\partial\gamma_l}\dot{\gamma}_l&=S_j.\end{aligned} \tag{15}$$

Since $\rho_3=\eta_3=0$, we can immediately solve for $\mathcal{Q}_{31}$ and $\mathcal{Q}_{32}$

$$\begin{aligned}\mathcal{Q}_{31}\rho_1+\mathcal{Q}_{32}\rho_2&=0\\ \mathcal{Q}_{31}\eta_1+\mathcal{Q}_{32}\eta_2&=S_3.\end{aligned} \tag{16}$$

$\rho_1\eta_2-\rho_2\eta_1=h$, as can be seen by direct calculation or can be more easily verified by recalling that $\boldsymbol{\varrho}$ and $\boldsymbol{\eta}$ are the same functions of h, β_1, β_2, and θ as in the two-body problem, and therefore $|\boldsymbol{\varrho}+\boldsymbol{\eta}|=h$.

Solving Eq. (16), we obtain

$$\begin{aligned}\mathcal{Q}_{31}&=(-\rho_2/h)S_3\\ \mathcal{Q}_{32}&=(\rho_1/h)S_3.\end{aligned} \tag{17}$$

The remaining four equations can be written as

$$\begin{bmatrix}\rho_2 & \dfrac{2\mu\rho_1\rho}{h^3} & -\rho_1^2 & -\rho_1\rho_2\\ -\rho_1 & \dfrac{2\mu\rho_2\rho}{h^3} & -\rho_1\rho_2 & -\rho_2^2\\ \eta_2 & -\beta_2+\dfrac{\mu\rho_2}{h^2\rho} & 0 & -h\\ -\eta_1 & \beta_1-\dfrac{\mu\rho_1}{h^2\rho} & h & 0\end{bmatrix}\begin{bmatrix}\alpha\\ \dot{h}\\ \dot{\beta}_1\\ \dot{\beta}_2\end{bmatrix}=\begin{bmatrix}0\\ 0\\ S_1\\ S_2\end{bmatrix} \tag{18}$$

where

$$\alpha=\mathcal{Q}_{12}=-\mathcal{Q}_{21}.$$

The determinant of the 4×4 matrix turns out to be just $h\rho^2$; therefore, if $h\neq0$ and $\rho\neq0$ we can invert it. The inversion can be simplified by partitioning the matrix into four 2×2 matrices. However, a less direct but simpler method is the following: Multiplying out matrix equation (18), we obtain for two of the equations:

$$\alpha\rho_2+\rho_1\left[\frac{2\mu}{h^3}\rho\dot{h}-\rho_1\dot{\beta}_1-\rho_2\dot{\beta}_2\right]=0,$$

$$-\alpha\rho_1+\rho_2\left[\frac{2\mu}{h^3}\rho\dot{h}-\rho_1\dot{\beta}_1-\rho_2\dot{\beta}_2\right]=0.$$

Multiplying the first of these by ρ_2 and the second by $-\rho_1$ we obtain

$$\alpha(\rho_1^2+\rho_2^2)=0,$$

which implies if $\rho\neq0$ that

$$\alpha=0 \tag{19}$$

and

$$(2\mu/h^3)\rho\dot{h}-\rho_1\dot{\beta}_1-\rho_2\dot{\beta}_2=0. \tag{20}$$

The remaining equations are

$$\left(-\beta_2+\frac{\mu}{h^2}\frac{\rho_2}{\rho}\right)\dot{h}-h\dot{\beta}_2=S_1 \tag{21}$$

and

$$\left(\beta_1-\frac{\mu}{h^2}\frac{\rho_1}{\rho}\right)\dot{h}+h\dot{\beta}_1=S_2. \tag{22}$$

Then

$$(20)-\frac{\rho_2}{h}(21)+\frac{\rho_1}{h}(22)$$

gives

$$\left[\frac{\beta_1\rho_1+\beta_2\rho_2}{h}+\frac{\mu\rho}{h^3}\right]\dot{h}=\frac{1}{h}[-\rho_2S_1+\rho_1S_2].$$

But, the coefficient of $\dot{h}$ is just $1/h$. Therefore

$$\dot{h}=-\rho_2S_1+\rho_1S_2. \tag{23}$$

For convenience let us define the vector **B** with components

$$B_i=-\beta_i+(\mu/h^2)(\rho_i/\rho).$$

We obtain

$$\begin{aligned}\dot{\beta}_1&=\frac{1}{h}[S_2+B_1\dot{h}]=\frac{1}{h}[-\beta_1\rho_2S_1+(1+B_1\rho_1)S_2],\\ \dot{\beta}_2&=\frac{1}{h}[-S_1+B_2\dot{h}]=\frac{1}{h}[-(1+B_2\rho_2)S_1+B_2\rho_1S_2].\end{aligned} \tag{24}$$

Since any orthogonal matrix has only three independent elements, one could, if he wished, select three α_l and represent all the elements of $\mathcal{L}$ in terms of them. Moreover the α_l can be chosen in such a way that for a particular problem we are not near any singularities. However, for certain applications such as programming the equations for numerical integration with a digital computer, it seems more advantageous to leave the matrix $\mathcal{L}$ expressed in terms of nine quantities although we will have redundant information.

It will be noted that the elements h,β_1,β_2 and α_l are obtained by integration of expressions that have the perturbing mass as a factor. This is not so with the equation $\dot{\theta}=h/\rho^2$. Hence in applications it may be necessary to use a smaller interval for the integration of θ than for the elements.

It will also be noted that $h=0$ is a singular point of the differential equations, Eqs. (24). It can be shown (see Wintner's *Analytical Foundations of Celestial Mechanics*) that h approaches this singular point as the particle approaches the body at the origin if $\mathbf{F}=0$ while all other bodies do not participate in a collision. However, h can be zero; that is, the instantaneous conic can degenerate into a straight line, even if there is no collision at all. In the latter case $\boldsymbol{\beta}$ cannot remain finite since

$$\rho=\frac{1}{(\mu/h^2)+\beta_1\cos\theta+\beta_2\sin\theta}$$

implies that as h approaches zero and $\boldsymbol{\beta}$ remains finite, ρ approaches zero.

SUMMARY AND DISCUSSION

If we let

$$\mathcal{L}=\begin{bmatrix}p_x & q_x & r_x\\ p_y & q_y & r_y\\ p_z & q_z & r_z\end{bmatrix},$$

since

$$\mathcal{Q}=\frac{S_3}{h}\begin{bmatrix}0 & 0 & \rho_2\\ 0 & 0 & -\rho_1\\ -\rho_2 & \rho_1 & 0\end{bmatrix},$$

we have

$$\dot{\mathcal{L}}=\frac{S_3}{h}\begin{bmatrix}-r_x\rho_2 & r_x\rho_1 & p_x\rho_2-q_x\rho_1\\ -r_y\rho_2 & r_y\rho_1 & p_y\rho_2-q_y\rho_1\\ -r_z\rho_2 & r_z\rho_1 & p_z\rho_2-q_z\rho_1\end{bmatrix}.$$

And, collecting the remaining equations, we have

$$\dot{h}=-\rho_2S_1+\rho_1S_2,$$

$$\dot{\beta}_1=\frac{1}{h}[-B_1\rho_2S_1+(1+B_1\rho_1)S_2],$$

$$\dot{\beta}_2=\frac{1}{h}[-(1+B_2\rho_2)S_1+B_2\rho_1S_2],$$

$$\dot{\theta}=\frac{h}{\rho^2},\quad \theta=0 \text{ at } t=t_1,$$

where

$$B_i=-\beta_i+\frac{\mu\rho_i}{h^2\rho},\quad \frac{1}{\rho}=\frac{\mu}{h^2}+\beta_1\cos\theta+\beta_2\sin\theta,$$

$$\rho_1=\rho\cos\theta,\quad \rho_2=\rho\sin\theta,$$

and

$$S_i=\frac{\partial R}{\partial\rho_i}+\tilde{\mathcal{L}}_{ik}\frac{F_k}{m}.$$

In order to integrate this system, we need the initia values of $\mathcal{L}$ in addition to $\beta_1(t_1)$, $\beta_2(t_1)$, and $h(t_1)$. Thes can readily be found in terms of $v_i(t_1)$ and $\dot{v}_i(t_1)$. Th equations, of course, remain valid, no matter what coni section is traced by the unperturbed orbit, since

$$\frac{\partial R}{\partial\rho_i}=-\kappa\sum_{j=1}^{n-1}m_j\left[\frac{\rho_i-\rho_i^j}{|\boldsymbol{\varrho}-\boldsymbol{\varrho}^j|^3}+\frac{\rho_i^j}{|\boldsymbol{\varrho}^j|^3}\right],$$

where m_j is the mass of any one of the n bodies excep the M body and ρ_i^j are the components of the positio vector of the jth body. If we set $\mathbf{F}=0$, we can write

$$\dot{h}=-\kappa\sum_j m_j(\boldsymbol{\varrho}^j\times\boldsymbol{\varrho})_3\left[\frac{1}{|\boldsymbol{\varrho}-\boldsymbol{\varrho}^j|^3}-\frac{1}{|\boldsymbol{\varrho}^j|^3}\right],$$

$$h\mathcal{Q}_{13}=-\kappa\sum_j m_j(\boldsymbol{\varrho}^j\times\boldsymbol{\varrho})_1\left[\frac{1}{|\boldsymbol{\varrho}-\boldsymbol{\varrho}^j|^3}-\frac{1}{|\boldsymbol{\varrho}^j|^3}\right],\qquad (2$$

$$h\mathcal{Q}_{23}=-\kappa\sum_j m_j(\boldsymbol{\varrho}^j\times\boldsymbol{\varrho})_2\left[\frac{1}{|\boldsymbol{\varrho}-\boldsymbol{\varrho}^j|^3}-\frac{1}{|\boldsymbol{\varrho}^j|^3}\right],$$

since $\rho_3=0$.

We can also write

$$\dot{\boldsymbol{\beta}}=-\frac{\kappa}{h}\sum_{i} m_j\left\{\left[\frac{1}{|\boldsymbol{\varrho}-\boldsymbol{\varrho}^j|^3}-\frac{1}{|\boldsymbol{\varrho}^j|^3}\right][(\boldsymbol{\varrho}^j\times\boldsymbol{\varrho})\cdot\mathbf{k}]\right.$$
$$\left.\times\mathbf{B}-\mathbf{k}\times\left[\frac{\boldsymbol{\varrho}-\boldsymbol{\varrho}^j}{|\boldsymbol{\varrho}-\boldsymbol{\varrho}^j|^3}+\frac{\boldsymbol{\varrho}^j}{|\boldsymbol{\varrho}^j|^3}\right]\right\}, \quad (26)$$

where $\boldsymbol{\beta}$ is the vector with components β_1, β_2, and 0, and $\mathbf{k}$ is a unit vector in the direction of $\boldsymbol{\varrho}+\boldsymbol{\eta}$.

It is interesting to note that if in addition to the M body there is only one other body, i.e., $j=1$ is the only term in the sums that occur in Eqs. (25) and (26), then Eq. (25) suggests the two Lagarangian solutions. For the collinear solution $\boldsymbol{\varrho}^1\times\boldsymbol{\varrho}=0$, while in the equilateral solution $(1/|\boldsymbol{\varrho}-\boldsymbol{\varrho}^j|^3)-(1/|\boldsymbol{\varrho}^j|^3)=0$. In both cases Eq. (25) gives immediately $\mathcal{Q}=0$, $h=\text{const}$, and $\mathfrak{L}=\text{const}$. Let us assume that $|\boldsymbol{\varrho}^1|=|\boldsymbol{\varrho}|=|\boldsymbol{\varrho}^1-\boldsymbol{\varrho}|$, where $|\boldsymbol{\varrho}|$ is not necessarily a constant. Equation (26) becomes

$$\dot{\boldsymbol{\beta}}=\frac{\mathbf{k}\times\boldsymbol{\varrho}}{|\boldsymbol{\varrho}|^3}\frac{\kappa m_1}{h}$$

or

$$\frac{d\boldsymbol{\beta}}{d\theta}=\frac{\kappa m_1}{h^2}\frac{\mathbf{k}\times\boldsymbol{\varrho}}{|\boldsymbol{\varrho}|}.$$

Therefore

$$\boldsymbol{\beta}=\frac{\kappa m_1}{h^2}\frac{\boldsymbol{\varrho}}{|\boldsymbol{\varrho}|}+\mathbf{a},$$

or

$$\frac{1}{\rho}=\kappa\frac{(m+M+m_1)}{h^2}+c\cos\theta+d\sin\theta,$$

where c and d are components of the constant vector of integration $\mathbf{a}$.

Similarly we can obtain

$$\frac{1}{\rho^1}=\frac{\kappa(m+M+m_1)}{h_1^2}+c_1\cos\theta_1+d_1\sin\theta_1,$$

where h_1 is the angular momentum of the body denoted by $j=1$, etc. In addition we have $\dot{\theta}_1=(h_1/h)\dot{\theta}$ or $\theta_1=(h_1/h)\theta+\alpha$, where α is a constant of integration. We must now ensure that the constraints $|\boldsymbol{\varrho}^1|=|\boldsymbol{\varrho}|=|\boldsymbol{\varrho}^1-\boldsymbol{\varrho}|$ are not violated, i.e.,

$$\rho^2+(\rho^1)^2-2\rho\rho^1\cos(\theta-\theta_1)=\rho^2=(\rho^1)^2.$$

This implies that

$$\cos\left[\theta\left(1-\frac{h_1}{h}\right)+\alpha\right]=\frac{1}{2}.$$

Since the angle must be time independent, we obtain $h=h_1$ and $\alpha=\pm 60°$. The remaining condition $|\boldsymbol{\varrho}^1|=|\boldsymbol{\varrho}|$ gives

$$c=c_1\cos\alpha+d_1\sin\alpha,$$
$$d=-c_1\sin\alpha+d_1\cos\alpha,$$

or

$$c_1^2+d_1^2=c^2+d^2.$$

We note that both conic sections are the same except that one has been rotated relative to the other by 60°. It is amusing to observe that the situation changes if one views the motion of the two bodies and the particle from the center of mass of the system. For example, if the conic sections obtained in the "heliocentric system" are similar ellipses with eccentricity e then a simple calculation shows that in the center of mass system the two bodies and the particle trace out ellipses with eccentricity e which are not necessarily similar but may in fact have different major axes. The collinear Lagrangian solution may be treated in the same way.

The Escape of Stars from Clusters. V. The Basic Escape Rate

IVAN KING
University of Illinois Observatory, Urbana, Illinois

(Received December 27, 1959)

An idealized relaxation problem is solved in which encounters maintain a steady state in a velocity distribution with a high-velocity cutoff. In contrast to treatments previously given, the stars here have encounters with each other rather than with a separate group of stars having a fixed velocity distribution. A time-dependent scale factor must be introduced into the velocities in order to conserve energy. Previous treatments of the problem have allowed the stars under consideration to extract energy from the stars encountered and have therefore not represented a true steady-state condition. The escape rate may be affected by the increase in kinetic energy that results from the evolutionary contraction of the cluster, but it is possible for this energy to be introduced in a way that leaves the present result unaffected. The escape rate derived here is one and a half times that recently found by Spitzer and Härm (1958).

IN the escape of stars from an isolated cluster the basic mechanism is the relaxation of a velocity distribution as a result of stellar encounters. All estimates of the rate of escape of stars therefore contain a multiplicative factor that expresses this relaxation rate. The relaxation problem can be idealized as follows: If stars simply disappear from the distribution on reaching the escape velocity v_e, at what rate do stars disappear? The average escape velocity for all stars in the cluster is fixed by the virial theorem according to the equation

$$\langle v_e^2\rangle_{\mathrm{av}}=4\langle v^2\rangle_{\mathrm{av}}, \qquad (1)$$

where $\langle v^2\rangle_{\mathrm{av}}$ is the mean-square velocity for all stars in the cluster. Hence in the idealized relaxation problem we take v_e to be twice the root-mean-square magnitude of the velocity.

The first discussions, by Ambartsumian (1938) and Spitzer (1940), assumed that the escape rate would be the same as if during each time of relaxation a Maxwellian velocity distribution was produced intact and then subjected to the escape-velocity cutoff. Chandrasekhar (1943a,b) made possible a more refined analysis by applying the Fokker-Planck equation to describe the detailed changes that encounters produce in a velocity distribution. He then introduced (Chandrasekhar 1943c,d) the idea of looking for a steady-state solution, in which the velocity distribution remains constant except for the necessary change in its normalizing factor as stars escape. The Fokker-Planck treatment was improved by Cohen, Spitzer, and Routly (1950) and again by Rosenbluth, MacDonald, and Judd (1957). Spitzer and Härm (1958) have recently made use of these advances to improve upon Chandrasekhar's solution. They give the steady-state solution for the case in which the stars considered encounter a set of stars having a Gaussian velocity distribution.

The next step logically is to remove the Gaussian approximation from the stars encountered and let the stars under consideration encounter each other. To do this it is necessary to go back to the Fokker-Planck equation and express the encounter integrals in terms of an arbitrary velocity distribution for the sta encountered. For an arbitrary isotropic distributio Eqs. (31), (40), (41), (45), and (46) of Rosenbluth MacDonald, and Judd (1957) give for the change du to encounters:

$$\frac{1}{4\pi\Gamma_a}\frac{\partial f_a}{\partial t}=\frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{m_a}{m_b}\mathcal{L}_2 f_a+\frac{\mathcal{L}_4+v^3\mathcal{U}_1}{3v}\frac{\partial f_a}{\partial v}\right), \qquad (2$$

where

$$\mathcal{L}_2(v)\equiv\int_0^v v^2 f_b(v)dv, \qquad (3$$

$$\mathcal{L}_4(v)\equiv\int_0^v v^4 f_b(v)dv, \qquad (4$$

$$\mathcal{U}_1(v)\equiv\int_v^\infty v f_b(v)dv. \qquad (5$$

Here $f_a(v)$ is the velocity distribution of the stars unde consideration, with respect to the velocity-space volum element $4\pi v^2dv$ and per unit volume of position space and $f_b(v)$ is the velocity distribution of the stars er countered. The masses of individual stars from eac distribution are m_a and m_b, respectively. Γ_a is a constan whose value Rosenbluth, MacDonald, and Judd giv only for the case of electrostatic interactions. For gravi tational interactions its value is

$$\Gamma_a=4\pi G^2 m_b^2\ln\alpha. \qquad (6$$

Spitzer and Härm (1958) have shown that in a sta cluster the cut-off parameter, α, is equal to $n/2$, wher n is the number of stars in the cluster.

In the present case the distributions f_a and f_b ar identical, so the equation becomes a second-degre equation. We seek a steady-state solution in which th shape of the distribution is independent of time. In it most general form, such a distribution will contain time-dependent scale factor for f and another time dependent scale factor for v. This latter scale change i needed in order to satisfy the principle of conservatio f energy. It did not appear in the solutions of Chandrasekhar and of Spitzer and Härm, because in hose solutions the stars under consideration were able o extract energy from the reservoir implied in the ime-independent velocity distribution of stars encountered. The scaling down of velocities in the present ase is analogous to the cooling that results from diabatic evaporation. The present case differs, however, in that the cut-off velocity is also scaled down, n order to maintain the relation [Eq. (1)] between the ut-off and the mean-square velocity.

The desired solution can thus be written in the form

$$f(v,t)=h(t)g(u), \tag{7}$$

where

$$v=u\cdot a(t). \tag{8}$$

Substitution in Eqs. (2)–(5), with $f_a\equiv f_b\equiv f$ and $m_a=m_b$, gives

$$\frac{1}{4\pi\Gamma h}\left(\frac{h'}{h}u^2g-\frac{a'}{a}u^3\frac{dg}{du}\right)=\frac{d}{du}\left(L_2g+\frac{L_4+u^3U_1}{3u}\frac{dg}{du}\right), \tag{9}$$

where

$$L_2(u)\equiv\int_0^u u^2g(u)du, \tag{10}$$

$$L_4(u)\equiv\int_0^u u^4g(u)du, \tag{11}$$

$$U_1(u)\equiv\int_u^\infty ug(u)du. \tag{12}$$

Primes denote differentiation with respect to time, and the subscript a has been dropped from Γ. Equation (9) is separable in u and t only if

$$a'/a=q(h'/h), \tag{13}$$

where q is a constant, for only then can the time-dependent terms be factored out of the left side of Eq. (9). This causes no excessive restriction; it merely determines how the conservation of energy causes the velocity scale to shrink.

The separate equations then become

$$h'=-4\pi\Gamma\mu h^2, \tag{14}$$

$$\frac{d}{du}\left(L_2g+\frac{L_4+u^3U_1}{3u}\frac{dg}{du}\right)=-\mu\left(u^2g-qu^3\frac{dg}{du}\right), \tag{15}$$

where μ is a constant. The solution of the time equation is

$$(1/h)-(1/h_0)=4\pi\Gamma\mu(t-t_0), \tag{16}$$

but the specific time dependence should not be taken too seriously, since the evolution of the cluster will be modified by other dynamical effects, such as the contraction of the cluster. What is important is the escape rate that is required for a steady state. This is determined by the eigenvalue solution of Eq. (15).

One formal integration can be performed immediately, resulting in

$$\frac{L_4+u^3U_1}{3u}\frac{dg}{du}+L_2[g+(1+3q)\mu]-q\mu u^3g=0. \tag{17}$$

The constant q can now be determined by applying the principle of conservation of energy. The total number of stars in the distribution per unit volume of space is

$$N=\int_0^{v_e} f\cdot 4\pi v^2dv$$
$$=4\pi ha^3L_2(u_e), \tag{18}$$

where v_e is the escape velocity and u_e the corresponding value of u. The integration might just as well have been extended to infinity, for f is zero beyond v_e. The total kinetic energy is

$$T=\int_0^{v_e}\tfrac{1}{2}mv^2f\cdot 4\pi v^2dv$$
$$=2\pi mha^5L_4(u_e). \tag{19}$$

This quantity will decrease with time, since each escaping star carries with it a kinetic energy $\frac{1}{2}mv_e^2$. Thus,

$$\frac{dT}{dt}=\tfrac{1}{2}mv_e^2\frac{dN}{dt}. \tag{20}$$

Comparison of Eqs. (18)–(20) shows that

$$\frac{1}{T}\frac{dT}{dt}=\frac{u_e^2L_2(u_e)}{L_4(u_e)}\frac{1}{N}\frac{dN}{dt}, \tag{21}$$

while differentiation of Eqs. (18) and (19) gives

$$\frac{1}{N}\frac{dN}{dt}=\frac{h'}{h}+3\frac{a'}{a}, \tag{22}$$

$$\frac{1}{T}\frac{dT}{dt}=\frac{h'}{h}+5\frac{a'}{a}. \tag{23}$$

The constants in Eq. (17) can now be identified. From Eqs. (13), (14), and (22),

$$(1+3q)\mu=-\frac{1}{4\pi\Gamma h}(1+3q)\frac{h'}{h}$$
$$=-\frac{1}{4\pi\Gamma h}\left(\frac{h'}{h}+3\frac{a'}{a}\right)$$
$$=-\frac{1}{4\pi\Gamma h}\frac{1}{N}\frac{dN}{dt}, \tag{24}$$

while

$$q\mu = -\frac{1}{4\pi\Gamma h}\frac{a'}{a}. \tag{25}$$

Solution of Eqs. (22) and (23) for a'/a, followed by application of Eq. (21), then gives

$$q\mu = -\frac{1}{4\pi\Gamma h}\cdot\frac{1}{2}\left(\frac{1}{T}\frac{dT}{dt} - \frac{1}{N}\frac{dN}{dt}\right)$$

$$= -\frac{1}{2}\left[\frac{u_e^2 L_2(u_e)}{L_4(u_e)} - 1\right]\frac{1}{4\pi\Gamma h}\frac{1}{N}\frac{dN}{dt}. \tag{26}$$

The first term in the brackets is just the square of the escape velocity divided by the mean-square velocity; by Eq. (1) this is equal to 4. Thus

$$q\mu = -\frac{3}{2}\frac{1}{4\pi\Gamma h}\frac{1}{N}\frac{dN}{dt}; \tag{27}$$

and Eq. (17) can be written

$$\frac{L_4 + u^3 U_1}{3u}\frac{dg}{du} + L_2(g+\lambda) - \tfrac{3}{2}\lambda u^3 g = 0, \tag{28}$$

where

$$\lambda = -\frac{1}{4\pi\Gamma h}\frac{1}{N}\frac{dN}{dt}. \tag{29}$$

Solution of this second-degree integrodifferential equation requires four integrations. Four boundary conditions can be applied at $u=0$: Both L_2 and L_4 must be zero there, while any positive values may be taken for $g(0)$ and $U_1(0)$, since they merely determine the scales of g and u. Two more boundary conditions must be satisfied at u_e, which is the value of u at which g first reaches zero. One is that

$$u_e^2 = \frac{4L_4(u_e)}{L_2(u_e)}; \tag{30}$$

in other words, that Eq. (1) be satisfied. This condition can be met by suitable choice of the eigenvalue λ. The remaining condition is that $U_1(u_e)=0$, since physically g must be zero for all higher values of u. This condition turns out to be satisfied automatically. The reason is that Eq. (17) had not one, but two, adjustable constants; fixing the relation between them to conserve energy leaves Eq. (29) with only one eigenvalue apparent, but the relation between the coefficients ensures the satisfaction of this one more condition.

Finding the correct value of λ thus involves choosing a trial value of λ, integrating Eq. (28) from $u=0$ to u_e, checking Eq. (30), and choosing a new trial value of λ. This iterative procedure was carried out automatically on the Illiac, the automatic digital computer at the University of Illinois. The final solution is given in Table I, normalized to $g(0)=1$, $u_e=2$. The last column compares $g(u)$ with a Gaussian curve that has a identical $\langle v^2\rangle_{\text{av}}$. The shape of $g(u)$ is very similar to tha of the function derived by Spitzer and Härm (1958 when the abscissas are adjusted to make the cutof coincide. The principal difference in shape is that th function in Table I satisfies Eq. (1), while the functio given by Spitzer and Härm has a $\langle v^2\rangle_{\text{av}}$ that is only 93 as large. More important, however, is the fact that th eigenvalue leads to an escape rate that will be show below to differ noticeably from that given by Spitze and Härm.

Table I. Steady-state velocity distribution.

u	$g(u)$	$ge^{\frac{3}{2}u^2}$
0.0	1.00000	1.0000
0.1	0.98613	1.0011
0.2	0.94619	1.0047
0.3	0.88317	1.0108
0.4	0.80191	1.0194
0.5	0.70831	1.0306
0.6	0.60857	1.0443
0.7	0.50860	1.0607
0.8	0.41340	1.0797
0.9	0.32677	1.1013
1.0	0.25112	1.1254
1.1	0.18754	1.1517
1.2	0.13601	1.1794
1.3	0.09567	1.2070
1.4	0.06510	1.2314
1.5	0.04267	1.2470
1.6	0.02669	1.242
1.7	0.01562	1.193
1.8	0.00815	1.052
1.9	0.00322	0.724
2.0	0.00000	0.000
$\lambda=0.004283$	$[L_2(u_e)/u_e^3]=0.03302$	

The function given in Table I is the steady-stat velocity distribution for a group of stars encounterin each other, under the idealized circumstances describe in the opening paragraph. Unfortunately, these circum stances are not very close to those in a star cluste where the stars are distributed not only in velocity bu in position as well, and the escape velocity varies fro one point of the cluster to another. Even more seriou is the fact that in the present solution the root-mea square velocity decreases, whereas it is known that i an isolated cluster the evolutionary contraction cause the root-mean-square velocity to increase. As the cluste contracts, potential energy is converted into kineti energy faster than the escaping stars carry energy away In addition to the changes that encounters make i the velocity distribution, there are then further velocit changes caused by the readjustment in position space A full treatment of the relaxation of the velocity dis tribution in a star cluster will thus involve an equatio for the distribution in the full phase space—a formidabl problem.

There is, however, one idealized model in which th root-mean-square velocity increases but the steady state solution is exactly the one given above. This i the case in which the additional energy derived fro the contraction of the cluster is divided among the stars in such a way that each star has its kinetic energy ncreased by the same fraction. This additional velocity change can be expressed as follows:

$$\frac{\delta}{\delta t}(\tfrac{1}{2}mv^2)=k\cdot\tfrac{1}{2}mv^2, \tag{31}$$

or

$$\frac{\delta v}{\delta t}=\tfrac{1}{2}kv. \tag{32}$$

This is a systematic additional velocity change for all stars. It can be looked at as producing an additional symmetrical outward flux in velocity space. Application of the law of conservation of numbers to a spherical shell of radius v then shows that the additional change produced in f is

$$\left(\frac{\partial f}{\partial t}\right)_{\text{add}}=-\frac{1}{4\pi v^2}\frac{\partial}{\partial v}(4\pi v^2 f\cdot\tfrac{1}{2}kv)$$
$$=-\frac{1}{2}\left(\frac{k}{v^2}\right)\frac{\partial}{\partial v}(v^3 f). \tag{33}$$

Thus the parentheses on the right side of Eq. (2) nclude an extra term $-(1/8\pi\Gamma)kv^3f$. After substitution of Eqs. (7) and (8), the parentheses on the right side of Eq. (9) contain an extra term $-(1/8\pi\Gamma)(k/h)u^3g$. Since h is a function of t, the variables can be separated in Eq. (9) only if k/h is time independent. This condition fixes a specific time dependence for k.

The last term on the left side of Eq. (17) then becomes

$$-\left(q\mu+\frac{k}{8\pi\Gamma h}\right)u^3g.$$

The energy added must be taken into account in Eq. (20), which becomes

$$\frac{dT}{dt}=\tfrac{1}{2}mv_e^2\frac{dN}{dt}+kT. \tag{20a}$$

As a result Eq. (21) has a term $+k$ added to its right side, and Eqs. (26) and (27) become

$$q\mu=-\frac{1}{2}\left[\frac{u_e^2L_2(u_e)}{L_4(u_e)}-1\right]\frac{1}{4\pi\Gamma h}\frac{1}{N}\frac{dN}{dt}-\frac{k}{8\pi\Gamma h} \tag{26a}$$

$$q\mu+\frac{k}{8\pi\Gamma h}=-\frac{3}{2}\frac{1}{4\pi\Gamma h}\frac{1}{N}\frac{dN}{dt}. \tag{27a}$$

Thus substitution in the modified Eq. (17) produces the same equation [Eq. (28)].

It is therefore possible for the solution given in Table I to represent a steady-state velocity distribution in a cluster in which the contraction feeds energy into the motions of the stars, provided that the energy is added in this particular fashion—each star having its energy increased by the same fraction. The additional condition—that k vary proportionally to h—is satisfied automatically. The relative energy input k for the motions in an actual cluster will be proportional to the relative rate of loss of stars, $-(1/N)(dN/dt)$; and Eq. (29) shows that this is indeed proportional to h.

The invariance of the escape rate under such a velocity change is easily understood. The cut-off velocity is scaled up in exactly the same proportion as all the individual velocities, and the result is simply a scale change without any transformation in the shape of the distribution.

The escape rate must still be derived from the eigenvalue λ. According to Eq. (29) the escape rate is

$$-\frac{1}{N}\frac{dN}{dt}=4\pi\Gamma h\lambda. \tag{34}$$

The quantity h can be eliminated by using Eq. (18) and noting that

$$\langle v^2\rangle_{\text{av}}=\frac{\int_0^{v_e}v^2f\cdot 4\pi v^2dv}{\int_0^{v_e}f\cdot 4\pi v^2dv}$$
$$=\frac{a^2L_4(u_e)}{L_2(u_e)} \tag{35}$$
$$=a^2,$$

where we have used Eq. (30) and the fact that the adopted solution has $u_e=2$. The expression for h can then be written

$$4\pi h=\frac{N}{L_2(u_e)\langle v^2\rangle_{\text{av}}^{\frac{3}{2}}}, \tag{36}$$

and the escape rate becomes

$$-\frac{1}{N}\frac{dN}{dt}=\frac{\lambda}{L_2(u_e)}\cdot\frac{4\pi G^2m^2\ln\alpha}{\langle v^2\rangle_{\text{av}}^{\frac{3}{2}}}$$
$$=\left(\frac{32}{27}\right)^{\frac{1}{2}}\frac{\lambda}{L_2(u_e)}\frac{1}{T_R}, \tag{37}$$

where T_R is the reference time used by Spitzer and Härm (1958). With the values given in Table I, this becomes

$$-\frac{1}{N}\frac{dN}{dt}=\frac{1}{56.7\,T_R}. \tag{38}$$

For the same quantity Spitzer and Härm found $1/(88.0\,T_R)$.

TABLE II. Dependence of escape rate on cutoff.

b	$1/K$
2.5	1.98
3.0	7.14
3.4	17.3
3.7	31.9
4.0	56.7
4.3	98.1
4.6	166
5.2	458

The problem was also solved for several other values of the cut-off parameter

$$b=\frac{u_e^2L_2(u_e)}{L_4(u_e)}. \qquad (39)$$

From Eqs. (17), (24), (26), and (29) it follows that the equation to be solved is

$$\frac{L_4+u^3U_1}{3u}\frac{dg}{du}+L_2(g+\lambda)-\tfrac{1}{2}(b-1)\lambda u^3g=0. \qquad (40)$$

In the interpretation of λ, Eq. (35) must be replaced by

$$\langle v^2\rangle_{\mathrm{av}}=\frac{a^2u_e^2}{b}, \qquad (41)$$

which leads to

$$-\frac{1}{N}\frac{dN}{dt}=\left(\frac{32}{27}\right)^{\frac{1}{2}}\left(\frac{u_e^2}{b}\right)^{\frac{3}{2}}\frac{\lambda}{L_2(u_e)}\frac{1}{T_R}. \qquad (42)$$

Table II gives the results, where

$$-\frac{1}{N}\frac{dN}{dt}=\frac{K}{T_R}. \qquad (43)$$

It appears that numerically K is closely proportional to $b^{-7.5}$.

In the Spitzer-Härm solution the stars encounter the members of a separate group of stars. In the present solution they encounter each other, and the escape rate is higher by a factor 1.55. Such a large difference cannot be due to differing values of the encounter integrals, for $g(u)$ closely resembles the distribution function found by Spitzer and Härm. The difference arises instead from the additional term in Eq. (28), which had to be introduced here in order to maintain a constant total energy. The existence of such a term is not just a peculiarity of the present treatment; it is a physical necessity. Without a proper energy balance, a velocity distribution cannot be called steady state. A situation in which the stars under consideration extract energy from an additional unchanging group cannot be maintained indefinitely, so that the condition assumed in previous solutions does not represent a steady state.

Intuitively it is very reasonable that adiabatic contraction of the velocity distribution should increase the escape rate, for decreasing the cut-off velocity is equivalent to increasing the velocities of all the stars relative to the cutoff. This has the effect of an additional speedup for the stars, promoting their escape.

In conclusion it should be emphasized that the problem solved here is an abstract relaxation problem rather than the actual problem of ejection of stars from clusters. While the present result is probably preferable to previous calculations of relaxation rates, it is still far from a correct description of the relaxation process in a star cluster. The equation that actually describes the evolution of the velocity distribution in an isolated cluster is the Liouville equation with the Fokker-Planck terms added. Except at the center of the cluster the velocity distribution is no longer isotropic, so that there are two independent variables in velocity space besides the one in position space. The encounter integrals in the Fokker-Planck terms also become much more complicated, because of the anisotropy of the velocity distribution. Furthermore, there does not seem to be any straightforward way of separating the variables, since some of the terms involve the dependent variable to the first degree and others to the second degree. In fact, a strictly steady state simply cannot exist in a star cluster, since some aspects of the equilibrium depend explicitly on the total number of stars in the cluster, which keeps decreasing. The best hope seems to be to consider the cluster to pass through a succession of quasi-steady states, characterized by successively smaller values of n.

Meanwhile it seems reasonable to incorporate the present result into a scheme of approximations such as that described at the beginning of Paper I of this series (King 1958). It should be remembered, however, that the present result may be directly affected by the particular way in which the evolutionary contraction of the cluster speeds up the motion of the stars.

This project was supported by a grant from the National Science Foundation.

REFERENCES

Ambartsumian, V. A. 1938, *Scientific Writings of Leningrad State Univ.* **22**, 19.

Chandrasekhar, S. 1943a, *Revs. Modern Phys.* **15**, 1.

——. 1943b, *Astrophys. J.* **97**, 255.

——. 1943c, *Astrophys. J.* **97**, 263.

——. 1943d, *Astrophys. J.* **98**, 54.

Cohen, R. S., Spitzer, L., and Routly, P. McR. 1950, *Phys. Rev.* **80**, 230.

King, I. 1958, *Astron. J.* **63**, 109.

Rosenbluth, M. N., MacDonald, W. M., and Judd, D. L. 1957, *Phys. Rev.* **107**, 1.

Spitzer, L. 1940, *Monthly Notices Roy. Astron. Soc.* **100**, 396.

Spitzer, L. and Härm, R. 1958, *Astrophys. J.* **127**, 544.

THE ASTRONOMICAL JOURNAL VOLUME 65, NUMBER 3 APRIL, 1960

Three-Color Photometry of AO Cassiopeiae

Robert H. Koch*
Flower and Cook Observatory, Philadelphia, Pennsylvania

(Received December 18, 1959)

Numerous photoelectric observations of AO Cas have established a sudden change in the shape of the light curve and have placed the variable on the *U,B,V* system. A constant period of revolution has been proposed, but short-term departures from this period are possible. The variations outside eclipse have been determined for the new light curves and for several old ones.

1. INTRODUCTION

THE most recent studies of AO Cas are due to Abhyankar (1959) and Mannino (1959). These articles adequately summarize the past history of the binary.

2. OBSERVATIONS

From September 1955 to January 1956, and from September 1956 to January 1957, AO Cas was observed with the photoelectric photometer attached to the 36-in. reflector of the Steward Observatory. In all, 554 yellow, 554 blue, and 552 violet observations were obtained. All have been corrected for differential extinction using the coefficients given below. The observations are listed in Table I (a, b, and c), and the yellow and violet ones assembled onto light curves are shown in Figs. 1 and 2. For all these measures the comparison star was BD +47°50, and the check star was BD +51°62. These same stars were used by Wood (1948).

By the night of October 27/28, 1955, a sufficient number of observations had been obtained so that the shape of the light curve was reasonably well known. Observations of this night and subsequent dates reveal that the shape of the light curve suffered an abrupt change between October 26/27 and October 27/28 of 1955. Accordingly, the first observing season is subdivided into Intervals I and II. Observations of the second season are homogeneous, yet differ from those of the preceding intervals, so that this is termed Interval III. The changes in the light curves for each of the intervals are not simple vertical translations along the magnitude scales. It thus seems unlikely that changes in the brightness of the comparison star can have caused the discordances. Moreover, within each interval the light curves show satisfactorily small scatter so that large irregular fluctuations of the comparison star appear to be ruled out.

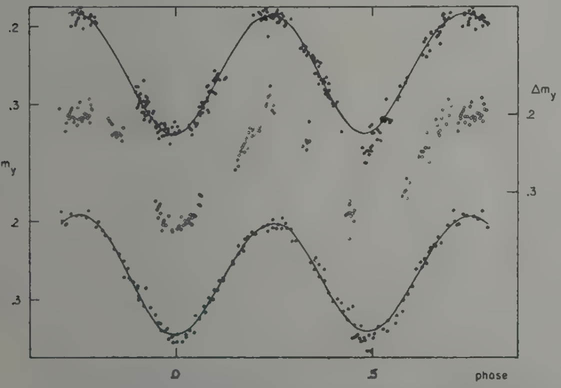


Fig. 1. From top to bottom, the yellow observations for Intervals I, II, and III. The curves were computed from the outside-eclipse variations given in Table IV and transformed to magnitude scale.

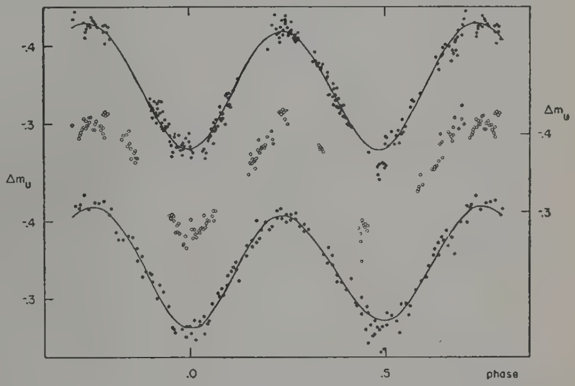


Fig. 2. From top to bottom the ultraviolet observations for Intervals I, II, and III. The curves were computed from the outside-eclipse variations given in Table IV and transformed to magnitude scale.

On each night BD +47°50 was checked at least once against BD +51°62. These observations were corrected for differential extinction using mean coefficients of $0^{\mathrm{m}}.220$ in yellow, $0^{\mathrm{m}}.353$ in blue, and $0^{\mathrm{m}}.683$ in violet. At each wavelength the seasonally averaged magnitude difference between the check and comparison stars shows a small change on the order of $0^{\mathrm{m}}.01$. For other stars on the program, seasonal differences proved to be negligible so that instrumental changes are unlikely.

On two nights the comparison stars were tied to the *U,B,V* system using 10 Lac and 50 And as standards. It had previously been noted that the color variation of AO Cas was quite small so that it was possible to observe the variable itself for the tie-in to the standard system. The reductions were accomplished by a least-squares solution, and the results are given in Table II.

---

* Now at Amherst College Observatory, Amherst, Massachusetts.

Table Ia. Yellow Observations of AO Cas.

V−C	JD ⊙ 2435000+	Phase
+0$^{\mathrm{m}}$326	372.7725	0.9701
0.339	.8273	0.9856
0.323	.8599	0.9949
0.329	.8751	0.9992
0.338	.8890	0.0032
0.186	373.7192	0.2388
0.185	.7414	0.2451
0.185	.7761	0.2549
0.180	.7893	0.2587
0.187	.8393	0.2729
0.194	.8532	0.2771
0.197	.8656	0.2803
0.190	.9135	0.2939
0.345	374.6894	0.5141
0.347	.7206	0.5230
0.321	.7533	0.5323
0.293	.7644	0.5354
0.318	.7783	0.5394
0.294	.8644	0.5638
0.289	.8783	0.5677
0.278	.8901	0.5711
0.273	.9311	0.5827
0.267	.9450	0.5867
0.292	376.6784	0.0786
0.312	.6978	0.0841
0.269	.7311	0.0936
0.256	.7429	0.0969
0.287	.7554	0.1005
0.268	.8353	0.1231
0.266	.8492	0.1271
0.245	377.6770	0.3620
0.247	.6971	0.3677
0.268	.7138	0.3725
0.266	.7291	0.3768
0.266	.7534	0.3837
0.274	.7687	0.3881
0.282	.7805	0.3914
0.302	.8353	0.4070
0.297	.8506	0.4113
0.335	.8839	0.4208
0.201	378.7687	0.6719
0.201	.7874	0.6772
0.192	.8132	0.6845
0.194	.8263	0.6882
0.189	.8548	0.6963
0.183	.8687	0.7002
0.174	.8798	0.7034
0.302	379.6805	0.9306
0.318	.7000	0.9362
0.312	.7125	0.9397
0.309	.7243	0.9431
0.328	.7430	0.9484
0.327	.7652	0.9547
0.325	.7847	0.9602
0.333	.8159	0.9691
0.332	.8298	0.9730
0.332	.8493	0.9786
0.334	.8611	0.9819
0.329	.8743	0.9856
0.330	.8875	0.9894
0.337	.9097	0.9957
0.339	.9229	0.9994
0.329	.9347	0.0028
0.335	.9465	0.0062
0.191	380.6896	0.2171
0.190	.7111	0.2232
0.187	.7229	0.2265
0.177	.7340	0.2297
0.213	.7535	0.2352
0.187	.7653	0.2385
0.188	.7798	0.2427
0.195	.8347	0.2582
+0.195	.8465	0.2616
+0$^{\mathrm{m}}$193	380.8597	0.2653
0.318	381.7688	0.5233
0.317	.7875	0.5287
0.320	.8354	0.5422
0.324	.8479	0.5458
0.321	.8597	0.5491
0.270	386.6765	0.9162
0.290	.6967	0.9219
0.292	.7085	0.9253
0.297	.7203	0.9287
0.300	.7508	0.9373
0.298	.7640	0.9410
0.322	.7821	0.9462
0.324	.7932	0.9493
0.335	.8272	0.9590
0.341	.8453	0.9641
0.341	.8571	0.9675
0.328	.8696	0.9710
0.339	.8883	0.9763
0.336	.9015	0.9801
0.351	.9133	0.9834
0.186	387.6622	0.1959
0.182	.7161	0.2112
0.184	.7363	0.2169
0.180	.7481	0.2203
0.189	.7592	0.2235
0.191	.7793	0.2292
0.182	.7911	0.2325
0.189	.8286	0.2432
0.190	.8376	0.2457
0.188	.8529	0.2501
0.185	.8620	0.2526
0.182	.8710	0.2552
0.185	.8856	0.2593
0.187	.8953	0.2621
0.190	.9043	0.2647
0.184	.9147	0.2676
0.195	.9356	0.2735
0.199	.9446	0.2761
0.328	388.7078	0.4927
0.327	.7252	0.4976
0.336	.7335	0.5000
0.337	.7418	0.5023
0.333	.7550	0.5061
0.337	.7634	0.5085
0.321	390.7051	0.0596
0.298	.7190	0.0635
0.303	.7273	0.0659
0.300	.7363	0.0684
0.298	.7453	0.0710
0.277	.7606	0.0753
0.296	.7724	0.0787
0.276	.7814	0.0812
0.281	.8183	0.0917
0.285	.8273	0.0943
0.267	.8419	0.0984
0.269	.8544	0.1019
0.266	.8634	0.1045
0.261	.8724	0.1071
0.269	.8814	0.1096
0.261	391.7440	0.3544
0.261	.7599	0.3589
0.274	.7690	0.3615
0.265	.7794	0.3645
0.273	.7891	0.3672
0.282	.8287	0.3785
0.286	.8384	0.3812
0.285	.8488	0.3842
0.291	.8592	0.3871
0.292	.8697	0.3901
0.293	.8801	0.3931
0.299	.9030	0.3995
+0.302	.9141	0.4027
+0$^{\mathrm{m}}$237	392.7086	0.6282
0.220	.7231	0.6323
0.216	.7322	0.6349
0.239	.7426	0.6378
0.224	.7572	0.6420
0.225	.7662	0.6445
0.231	.7745	0.6469
0.207	.7842	0.6496
0.208	.7940	0.6524
0.206	.8329	0.6635
0.214	.8419	0.6660
0.212	.8509	0.6686
0.207	.8606	0.6713
0.223	.8697	0.6739
0.188	.8877	0.6790
0.184	.8968	0.6816
0.200	.9051	0.6840
0.278	393.6565	0.8972
0.277	.6690	0.9008
0.301	.6773	0.9031
0.292	.6856	0.9055
0.287	.6940	0.9079
0.295	.7072	0.9116
0.304	.7155	0.9140
0.297	.7245	0.9165
0.297	.7329	0.9189
0.303	.7412	0.9213
0.300	.7544	0.9250
0.314	.7627	0.9274
0.310	.7711	0.9297
0.310	.7794	0.9321
0.322	.7877	0.9344
0.329	.8336	0.9475
0.335	.8419	0.9498
0.336	.8502	0.9522
0.344	.8586	0.9546
0.335	.8669	0.9569
0.356	395.6919	0.4749
0.363	.7051	0.4786
0.375	.7135	0.4810
0.375	.7218	0.4834
0.359	.7301	0.4857
0.359	.7433	0.4895
0.364	.7516	0.4918
0.355	.7669	0.4962
0.359	.7753	0.4985
0.353	.7836	0.5009
0.328	397.6600	0.0334
0.332	.6683	0.0358
0.334	.6767	0.0382
0.338	.6850	0.0405
0.321	.7010	0.0451
0.324	.7093	0.0474
0.329	.7183	0.0500
0.324	.7267	0.0524
0.327	.7350	0.0547
0.304	.7482	0.0585
0.315	.7565	0.0608
0.310	.7649	0.0632
0.311	.7732	0.0656
0.305	.7815	0.0679
0.308	.8246	0.0801
0.284	.8329	0.0825
0.277	.8426	0.0853
0.283	.8510	0.0876
0.293	.8593	0.0900
0.228	398.6544	0.3157
0.228	.6670	0.3192
0.228	.6746	0.3214
0.224	.6829	0.3237
0.231	.6912	0.3261
0.229	.7038	0.3297
+0.227	.7121	0.3320

TABLE Ia. (Continued)

V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase
+0$^{\mathrm{m}}$237	398.7204	0.3344	+0$^{\mathrm{m}}$327	409.6489	0.4425	+0$^{\mathrm{m}}$221	438.6509	0.6737
0.235	.7288	0.3368	0.221	410.6183	0.7177	0.205	.6578	0.6756
0.222	.7371	0.3391	0.210	.6308	0.7212	0.194	.6662	0.6780
0.238	.7676	0.3481	0.200	.6391	0.7236	0.212	.7058	0.6893
0.239	.7760	0.3502	0.205	.6475	0.7259	0.180	.7231	0.6942
0.242	.7843	0.3525	0.199	.6558	0.7283	0.201	.7328	0.6969
0.247	.7926	0.3549	0.195	.6690	0.7320	0.190	.7460	0.7007
0.252	.8288	0.3652	0.191	.6773	0.7344	0.311	439.6141	0.9471
0.268	.8420	0.3689	0.203	.6857	0.7368	0.322	.6280	0.9510
0.266	.8503	0.3713	0.210	.6940	0.7391	0.331	.6349	0.9530
0.273	.8600	0.3740	0.204	.7023	0.7415	0.319	.6418	0.9549
0.270	.8683	0.3764	0.215	.7155	0.7452	0.316	.6488	0.9569
0.272	.8767	0.3787	0.214	.7239	0.7476	0.327	.6578	0.9595
0.167	403.6322	0.7284	0.191	.7322	0.7500	0.349	.6731	0.9638
0.200	.6454	0.7322	0.206	.7405	0.7523	0.341	.6807	0.9660
0.185	.6538	0.7346	0.184	.7544	0.7563	0.339	.6891	0.9683
0.183	.6621	0.7369	0.204	.7628	0.7587	0.336	.6960	0.9703
0.175	.6704	0.7393	0.208	.7711	0.7610	0.343	.7057	0.9731
0.199	.6829	0.7428	0.199	.7996	0.7691	0.331	.7328	0.9807
0.194	.6913	0.7452	0.200	.8079	0.7715	0.349	.7439	0.9839
0.180	.6996	0.7476	0.205	.8211	0.7752	0.349	.7516	0.9861
0.190	.7079	0.7499	0.199	.8294	0.7776	0.351	.7592	0.9882
0.185	.7163	0.7523	0.202	.8378	0.7799	0.351	.7668	0.9904
0.176	.7288	0.7558	0.210	.8461	0.7823	0.188	440.6058	0.2285
0.182	.7371	0.7582	0.348	411.6190	0.0016	0.186	.6201	0.2326
0.183	.7454	0.7606	0.343	.6329	0.0056	0.177	.6288	0.2350
0.204	.7538	0.7629	0.348	.6412	0.0079	0.176	.6382	0.2377
0.190	.7621	0.7653	0.348	.6496	0.0103	0.166	.6479	0.2404
0.191	.8010	0.7763	0.331	.6593	0.0131	0.195	.6635	0.2449
0.188	.8093	0.7787	0.343	.6718	0.0166	0.188	.6729	0.2475
0.187	.8176	0.7810	0.344	.6815	0.0194	0.199	.6829	0.2504
0.199	.8260	0.7834	0.336	.6898	0.0217	0.179	449.5962	0.7801
0.194	.8350	0.7860	0.341	.7016	0.0251	0.187	.6115	0.7844
0.194	.8482	0.7897	0.339	.7100	0.0275	0.194	.6188	0.7865
0.189	.8565	0.7921	0.334	.7183	0.0298	0.194	.6358	0.7913
0.195	.8649	0.7945	0.340	.7343	0.0344	0.194	.6428	0.7933
0.196	.8732	0.7968	0.329	.7426	0.0367	0.243	452.6110	0.6357
0.284	407.7447	0.8596	0.342	.7523	0.0395	0.224	.6259	0.6400
0.305	.7572	0.8991	0.333	.7607	0.0419	0.225	.6329	0.6420
0.290	.7656	0.9015	0.329	.7697	0.0444	0.243	.6398	0.6439
0.293	.7739	0.9039	0.331	.7836	0.0484	0.262	.6474	0.6461
0.298	.7822	0.9062	0.315	.8148	0.0572	0.232	458.5818	0.3303
0.310	.7982	0.9108	0.303	.8232	0.0596	0.244	.5944	0.3339
0.296	.8065	0.9131	0.318	.8315	0.0620	0.236	.6013	0.3359
0.311	.8162	0.9159	0.305	.8398	0.0643	0.209	.6082	0.3378
0.320	.8246	0.9183	0.306	413.6447	0.5766	0.234	.6172	0.3404
0.315	.8343	0.9210	0.300	.6613	0.5813	0.316	476.5855	0.4400
0.272	408.6176	0.1498	0.298	.6697	0.5837	0.312	.5980	0.4436
0.257	.6308	0.1536	0.283	.6808	0.5868	0.330	.6050	0.4456
0.261	.6392	0.1560	0.308	.6877	0.5888	0.328	.6119	0.4475
0.266	.6475	0.1583	0.289	.7092	0.5949	0.328	.6202	0.4499
0.252	.6558	0.1607	0.254	.7968	0.6197	0.337	.6334	0.4536
0.247	.6683	0.1642	0.247	.8134	0.6244	0.323	.6404	0.4556
0.240	.6767	0.1666	0.256	.8231	0.6272	0.266	732.6751	0.1252
0.242	.6850	0.1690	0.249	.8336	0.6302	0.251	.7424	0.1443
0.240	.6933	0.1713	0.204	428.6207	0.8270	0.231	.7890	0.1575
0.248	.7017	0.1737	0.208	.6332	0.8305	0.238	.8313	0.1695
0.237	.7142	0.1772	0.210	.6430	0.8333	0.222	.8556	0.1764
0.231	.7225	0.1796	0.213	.6541	0.8365	0.227	734.6213	0.6775
0.243	.7308	0.1820	0.226	.6610	0.8384	0.204	.6748	0.6927
0.224	.7392	0.1843	0.225	.6721	0.8416	0.186	.7272	0.7076
0.217	.7489	0.1871	0.224	.6853	0.8453	0.203	.7734	0.7207
0.228	.7648	0.1916	0.228	.6957	0.8483	0.190	.8109	0.7313
0.220	.7732	0.1940	0.224	.7013	0.8499	0.192	.8543	0.7437
0.219	.7815	0.1963	0.224	.7096	0.8522	0.341	737.6450	0.5357
0.226	.7898	0.1987	0.227	.7284	0.8575	0.330	.7298	0.5597
0.221	.8239	0.2084	0.250	.7353	0.8595	0.320	.7384	0.5622
0.221	.8385	0.2125	0.234	.7430	0.8617	0.301	.7950	0.5782
0.216	.8440	0.2141	0.217	.7520	0.8642	0.293	.8402	0.5911
0.331	409.6121	0.4321	0.207	438.6113	0.6624	0.283	.8832	0.6033
0.328	.6246	0.4356	0.214	.6259	0.6666	0.218	738.6410	0.8184
0.317	.6329	0.4380	0.223	.6328	0.6686	0.222	.6826	0.8302
+0.361	.6412	0.4403	+0.227	.6426	0.6713	+0.236	.7260	0.8425

TABLE Ia. (Continued)

V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase
$+0\overset{\mathrm{m}}{.}242$	738.7639	0.8532	$+0\overset{\mathrm{m}}{.}277$	761.7492	0.3768	$+0\overset{\mathrm{m}}{.}300$	793.5808	0.4111
0.252	.8055	0.8650	0.292	.7909	0.3886	0.305	.6235	0.4232
0.261	.8486	0.8773	0.303	.8333	0.4007	0.312	.6780	0.4387
0.279	.8884	0.8886	0.193	766.6177	0.7586	0.310	.7225	0.4513
0.284	.9302	0.9004	0.196	.6569	0.7697	0.333	.7357	0.4550
0.330	742.6400	0.9533	0.187	.7013	0.7823	0.335	.7933	0.4714
0.341	.6716	0.9623	0.207	.7465	0.7951	0.354	.8322	0.4824
0.339	.7232	0.9769	0.201	.7611	0.7992	0.351	795.5821	0.9791
0.355	.7690	0.9899	0.212	.8062	0.8121	0.350	.6148	0.9884
0.345	.8086	0.0012	0.261	769.6194	0.6105	0.355	.6547	0.9997
0.355	.8523	0.0136	0.253	.6586	0.6216	0.353	.6971	0.0117
0.346	.8905	0.0244	0.247	.6996	0.6332	0.349	.7412	0.0242
0.336	.9332	0.0366	0.238	.7406	0.6449	0.340	.7783	0.0348
0.322	.9728	0.0478	0.229	.7822	0.6567	0.343	.8099	0.0437
0.206	750.6397	0.2238	0.219	.8246	0.6687	0.345	818.5727	0.5041
0.212	.6807	0.2354	0.349	772.5972	0.4556	0.338	.6554	0.5276
0.209	.7185	0.2461	0.350	.6378	0.4672	0.327	.6974	0.5395
0.204	.7654	0.2594	0.349	.6791	0.4789	0.316	.7384	0.5512
0.209	.8018	0.2698	0.346	.7222	0.4911	0.317	820.5803	0.0739
0.209	.8456	0.2822	0.349	.7621	0.5024	0.304	.6164	0.0842
0.225	.8876	0.2941	0.335	.8051	0.5146	0.288	.6563	0.0955
0.224	.9293	0.3060	0.247	782.8130	0.3550	0.276	.6994	0.1077
0.210	757.6426	0.2113	0.255	.8463	0.3645	0.275	820.7383	0.1188
0.207	.6797	0.2218	0.262	.8894	0.3767	0.258	829.5766	0.6272
0.204	.7214	0.2337	0.316	786.5858	0.4258	0.254	.6155	0.6382
0.197	.7638	0.2457	0.326	.6258	0.4371	0.234	.6547	0.6494
0.209	.8054	0.2575	0.351	.6709	0.4499	0.231	.6968	0.6613
0.195	.8475	0.2695	0.346	.7126	0.4618	0.223	.7263	0.6697
0.203	.8888	0.2812	0.355	.7494	0.4722	0.286	830.6144	0.9217
0.211	.9270	0.2920	0.367	.8025	0.4873	0.311	.6547	0.9332
0.325	760.6214	0.0567	0.367	.8355	0.4966	0.321	.6967	0.9451
0.310	.6676	0.0698	0.200	787.5862	0.7097	0.331	.7404	0.9575
0.300	.7107	0.0821	0.204	.6143	0.7177	0.286	837.5768	0.8978
0.301	.7530	0.0941	0.254	792.6471	0.1461	0.301	.5917	0.9020
0.289	.7954	0.1061	0.242	.6888	0.1579	0.297	.6004	0.9045
0.273	.8367	0.1178	0.232	.7305	0.1697	0.309	.6445	0.9170
0.253	761.6298	0.3429	0.220	.7714	0.1814	0.272	858.6072	0.8665
0.261	.6652	0.3530	0.214	.8128	0.1931	+0.273	.6531	0.8795
+0.271	.7083	0.3652	+0.216	.8506	0.2038			

This table also lists the probable errors for each color and magnitude and the number of observations for each star. With the colors and magnitudes of BD +47°50 established, the light amplitude of AO Cas can be determined on the *V* scale. This is given for the light curve of the second observing season. Fortunately, it is possible to check the photometry against independent observations. Hiltner (1956) has observed the check star, and AO Cas was observed by Hiltner and Johnson (1956) for their investigation of the law of interstellar reddening. Their results appear as entries 3 and 5 of Table II.

Errors for the colors and magnitudes observed by Hiltner and Johnson are not known, but the systematic errors from one of their programs to another do not exceed $0\overset{\mathrm{m}}{.}01$. It is probable that the observational errors for their results are not greater than the values determined at Tucson.

For BD +51°62, the color determinations agree satisfactorily. The *V* magnitudes, however, show a discordance greater than expected. This result, together with the seasonal changes between the comparison and check stars, suggests that BD +51°62 shows a small light variation. Hiltner (1956) lists this star as B5IV. Whether or not it shows periodic variations similar to some early B-type variables cannot be determined from these observations.

The violet index of AO Cas seems fairly well established. Discrepancies do exist, however, in the values of *V* and *B-V*. It should be noted that the colors of AO Cas given by Hiltner and Johnson remove the binary rather far from their mean reddening line. With the Tucson colors, on the other hand, AO Cas falls on the mean line. It is possible then that the single *V* observation of Hiltner and Johnson either is in error or happened to catch AO Cas at a time when it was uncommonly yellow. Since it is known that the light curve does change, this latter possibility cannot be ignored.

3. EPOCH AND PERIOD

Because the orbital period of AO Cas is relatively long, the determination of times of minimum light is best accomplished by centering at $0\overset{\mathrm{P}}{.}0$ the bottom of primary eclipse in a seasonal light curve. It frequently happens that a night's observations terminate just before minimum light or begin just after it. Extra-

TABLE Ib. Blue Observations of AO Cas.

V−C	JD ⊙ 2435000+	Phase
+0$^{\mathrm{m}}$308	372.8286	0.9860
0.297	.8513	0.9925
0.294	.8765	0.9996
0.306	.8904	0.0035
0.149	373.7206	0.2392
0.153	.7428	0.2455
0.147	.7789	0.2557
0.158	.7907	0.2591
0.163	.8420	0.2736
0.164	.8545	0.2772
0.161	.8670	0.2807
0.180	.9149	0.2943
0.288	374.6908	0.5145
0.311	.7220	0.5234
0.289	.7547	0.5327
0.282	.7658	0.5358
0.272	.7797	0.5398
0.258	.8672	0.5646
0.249	.8797	0.5681
0.240	.8915	0.5715
0.244	.9325	0.5831
0.234	.9464	0.5871
0.267	376.6797	0.0790
0.269	.6992	0.0845
0.240	.7325	0.0940
0.223	.7450	0.0975
0.238	.7568	0.1009
0.219	.8367	0.1235
0.217	.8520	0.1279
0.203	377.6784	0.3624
0.195	.6985	0.3681
0.224	.7159	0.3731
0.214	.7305	0.3772
0.241	.7548	0.3841
0.240	.7700	0.3884
0.243	.7819	0.3918
0.265	.8367	0.4074
0.270	.8527	0.4119
0.275	.8877	0.4218
0.162	378.7701	0.6723
0.155	.7888	0.6776
0.153	.8145	0.6849
0.164	.8277	0.6886
0.149	.8583	0.6973
0.144	.8701	0.7006
0.139	.8812	0.7038
0.275	379.6819	0.9310
0.269	.7014	0.9366
0.270	.7139	0.9401
0.275	.7257	0.9435
0.275	.7444	0.9488
0.274	.7666	0.9551
0.282	.7861	0.9606
0.279	.8173	0.9695
0.291	.8312	0.9734
0.302	.8507	0.9790
0.297	.8625	0.9823
0.297	.8757	0.9860
0.292	.8889	0.9898
0.298	.9111	0.9961
0.297	.9243	0.9998
0.295	.9361	0.0032
0.297	.9479	0.0066
0.155	380.6910	0.2175
0.146	.7125	0.2236
0.156	.7243	0.2269
0.146	.7361	0.2303
0.159	.7548	0.2356
0.165	.7667	0.2389
0.158	.7812	0.2431
0.160	.8361	0.2586
0.165	.8479	0.2620
+0.152	.8611	0.2657
+0$^{\mathrm{m}}$290	381.7702	0.5237
0.279	.7889	0.5291
0.279	.8368	0.5426
0.269	.8493	0.5462
0.274	.8611	0.5495
0.258	386.6779	0.9166
0.260	.6981	0.9223
0.260	.7099	0.9257
0.268	.7217	0.9290
0.261	.7404	0.9343
0.269	.7522	0.9377
0.271	.7654	0.9414
0.280	.7835	0.9466
0.279	.7946	0.9497
0.287	.8286	0.9594
0.294	.8467	0.9645
0.298	.8585	0.9679
0.288	.8710	0.9714
0.292	.8897	0.9767
0.293	.9029	0.9805
0.299	.9147	0.9838
0.181	387.6675	0.1974
0.150	.7175	0.2116
0.165	.7376	0.2173
0.153	.7495	0.2207
0.165	.7606	0.2239
0.166	.7807	0.2296
0.154	.7925	0.2329
0.162	.8293	0.2434
0.145	.8384	0.2460
0.153	.8536	0.2503
0.156	.8626	0.2528
0.152	.8717	0.2554
0.153	.8863	0.2595
0.151	.8960	0.2623
0.159	.9050	0.2649
0.153	.9154	0.2678
0.153	.9363	0.2737
0.164	.9453	0.2763
0.295	388.7085	0.4929
0.292	.7259	0.4978
0.289	.7342	0.5002
0.295	.7425	0.5025
0.296	.7557	0.5063
0.304	.7641	0.5087
0.286	390.7058	0.0598
0.256	.7196	0.0637
0.266	.7280	0.0661
0.268	.7370	0.0686
0.260	.7460	0.0712
0.235	.7613	0.0755
0.253	.7731	0.0789
0.247	.7822	0.0815
0.238	.8190	0.0919
0.239	.8280	0.0945
0.242	.8426	0.0986
0.242	.8551	0.1021
0.239	.8641	0.1047
0.225	.8731	0.1073
0.238	.8822	0.1098
0.231	391.7447	0.3546
0.232	.7606	0.3591
0.225	.7697	0.3617
0.232	.7801	0.3647
0.234	.7898	0.3674
0.244	.8294	0.3787
0.247	.8391	0.3814
0.252	.8495	0.3844
0.254	.8599	0.3873
0.252	.8704	0.3903
0.260	.8808	0.3932
0.257	.9037	0.3997
+0.256	.9148	0.4029
+0$^{\mathrm{m}}$190	392.7092	0.6284
0.186	.7238	0.6325
0.176	.7329	0.6351
0.188	.7433	0.6380
0.185	.7579	0.6422
0.182	.7669	0.6447
0.185	.7752	0.6471
0.177	.7849	0.6498
0.172	.7947	0.6526
0.169	.8336	0.6637
0.171	.8426	0.6662
0.176	.8516	0.6688
0.181	.8613	0.6715
0.175	.8704	0.6741
0.164	.8884	0.6792
0.178	.8974	0.6818
0.175	.9058	0.6842
0.246	393.6572	0.8974
0.248	.6697	0.9010
0.263	.6780	0.9033
0.249	.6863	0.9057
0.249	.6947	0.9081
0.253	.7079	0.9118
0.258	.7162	0.9142
0.256	.7252	0.9167
0.264	.7336	0.9191
0.264	.7419	0.9215
0.265	.7551	0.9252
0.267	.7634	0.9276
0.272	.7718	0.9299
0.277	.7801	0.9323
0.271	.7884	0.9346
0.283	.8343	0.9477
0.299	.8426	0.9500
0.296	.8509	0.9524
0.304	.8593	0.9548
0.297	.8676	0.9571
0.313	395.6926	0.4751
0.320	.7058	0.4788
0.330	.7142	0.4812
0.334	.7225	0.4836
0.322	.7308	0.4859
0.318	.7440	0.4897
0.315	.7523	0.4920
0.324	.7676	0.4964
0.322	.7760	0.4987
0.318	.7843	0.5011
0.290	397.6607	0.0336
0.290	.6690	0.0360
0.294	.6774	0.0384
0.290	.6857	0.0407
0.289	.7017	0.0453
0.294	.7100	0.0476
0.286	.7190	0.0502
0.285	.7274	0.0526
0.284	.7357	0.0549
0.271	.7489	0.0587
0.274	.7572	0.0610
0.260	.7656	0.0634
0.262	.7739	0.0658
0.264	.7822	0.0681
0.263	.8253	0.0803
0.249	.8336	0.0827
0.241	.8433	0.0855
0.245	.8517	0.0878
0.248	.8600	0.0902
0.178	398.6551	0.3159
0.195	.6676	0.3194
0.189	.6753	0.3216
0.185	.6836	0.3239
0.187	.6920	0.3263
0.195	.7044	0.3298
+0.193	.7128	0.3322

TABLE Ib. (Continued)

V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase
+0$^{\mathrm{m}}$197	398.7211	0.3346	+0$^{\mathrm{m}}$282	409.6496	0.4427	+0$^{\mathrm{m}}$181	438.6516	0.6739
0.196	.7294	0.3369	0.174	410.6190	0.7178	0.161	.6586	0.6759
0.193	.7378	0.3393	0.158	.6315	0.7214	0.147	.6669	0.6782
0.206	.7683	0.3480	0.167	.6398	0.7238	0.171	.7065	0.6895
0.211	.7767	0.3504	0.173	.6482	0.7261	0.140	.7238	0.6944
0.216	.7850	0.3527	0.167	.6565	0.7285	0.168	.7336	0.6972
0.212	.7933	0.3551	0.162	.6697	0.7322	0.158	.7467	0.7009
0.217	.8294	0.3653	0.159	.6780	0.7346	0.279	439.6148	0.9473
0.242	.8426	0.3691	0.170	.6864	0.7370	0.281	.6286	0.9512
0.234	.8510	0.3715	0.170	.6947	0.7393	0.285	.6356	0.9532
0.238	.8607	0.3742	0.167	.7030	0.7417	0.277	.6425	0.9551
0.233	.8690	0.3766	0.162	.7162	0.7454	0.276	.6495	0.9571
0.234	.8774	0.3789	0.166	.7246	0.7478	0.284	.6585	0.9597
0.118	403.6329	0.7286	0.158	.7329	0.7502	0.291	.6738	0.9640
0.158	.6461	0.7324	0.171	.7412	0.7525	0.295	.6814	0.9662
0.150	.6545	0.7348	0.166	.7551	0.7565	0.287	.6898	0.9685
0.149	.6628	0.7371	0.161	.7634	0.7588	0.280	.6967	0.9705
0.153	.6711	0.7395	0.160	.7718	0.7612	0.296	.7064	0.9733
0.152	.6836	0.7430	0.170	.8002	0.7693	0.303	.7335	0.9809
0.145	.6920	0.7454	0.165	.8086	0.7717	0.306	.7446	0.9841
0.143	.7003	0.7478	0.164	.8218	0.7754	0.303	.7523	0.9863
0.139	.7086	0.7501	0.172	.8301	0.7778	0.303	.7599	0.9884
0.143	.7170	0.7525	0.160	.8384	0.7801	0.297	.7675	0.9906
0.150	.7295	0.7560	0.171	.8468	0.7825	0.151	440.6062	0.2286
0.154	.7378	0.7584	0.285	411.6197	0.0018	0.156	.6204	0.2326
0.154	.7461	0.7608	0.300	.6336	0.0058	0.145	.6291	0.2351
0.150	.7545	0.7631	0.302	.6419	0.0081	0.146	.6385	0.2378
0.155	.7628	0.7655	0.298	.6502	0.0105	0.144	.6482	0.2405
0.150	.8017	0.7765	0.288	.6600	0.0133	0.150	.6638	0.2450
0.148	.8100	0.7789	0.292	.6725	0.0168	0.140	.6732	0.2476
0.143	.8184	0.7813	0.304	.6822	0.0196	0.153	.6833	0.2505
0.158	.8267	0.7836	0.304	.6905	0.0219	0.144	449.5969	0.7803
0.150	.8357	0.7862	0.299	.7023	0.0253	0.152	.6122	0.7846
0.146	.8489	0.7899	0.297	.7107	0.0277	0.161	.6195	0.7867
0.149	.8572	0.7923	0.287	.7190	0.0300	0.160	.6365	0.7915
0.152	.8656	0.7947	0.278	.7350	0.0346	0.151	.6434	0.7935
0.165	.8739	0.7970	0.282	.7433	0.0369	0.201	452.6117	0.6359
0.242	407.7454	0.8958	0.302	.7530	0.0397	0.190	.6266	0.6402
0.250	.7579	0.8993	0.280	.7614	0.0421	0.186	.6336	0.6422
0.253	.7662	0.9017	0.285	.7704	0.0446	0.208	.6405	0.6441
0.260	.7746	0.9041	0.279	.7843	0.0486	0.208	.6482	0.6463
0.256	.7829	0.9064	0.271	.8155	0.0574	0.206	458.5825	0.3305
0.268	.7989	0.9110	0.263	.8238	0.0598	0.199	.5950	0.3341
0.265	.8072	0.9133	0.279	.8322	0.0622	0.197	.6020	0.3361
0.268	.8170	0.9161	0.269	.8405	0.0645	0.195	.6089	0.3380
0.280	.8253	0.9185	0.267	413.6454	0.5768	0.200	.6180	0.3406
0.264	.8350	0.9212	0.256	.6620	0.5815	0.275	476.5862	0.4402
0.225	408.6183	0.1500	0.252	.6704	0.5839	0.274	.5987	0.4438
0.200	.6315	0.1538	0.244	.6787	0.5862	0.290	.6057	0.4458
0.213	.6398	0.1561	0.230	.6905	0.5896	0.287	.6126	0.4477
0.214	.6482	0.1585	0.246	.7099	0.5951	0.287	.6209	0.4501
0.205	.6565	0.1609	0.217	.7974	0.6199	0.297	.6341	0.4538
0.196	.6690	0.1644	0.218	.8141	0.6246	0.289	.6411	0.4558
0.199	.6774	0.1668	0.215	.8238	0.6274	0.236	732.6754	0.1252
0.215	.6857	0.1692	0.211	.8342	0.6303	0.211	.7428	0.1444
0.199	.6940	0.1715	0.177	428.6214	0.8272	0.202	.7893	0.1576
0.200	.7024	0.1739	0.167	.6339	0.8307	0.202	.8317	0.1696
0.192	.7148	0.1774	0.166	.6436	0.8335	0.186	.8560	0.1765
0.203	.7232	0.1798	0.173	.6548	0.8366	0.183	734.6210	0.6774
0.195	.7315	0.1822	0.180	.6617	0.8386	0.158	.6744	0.6926
0.183	.7398	0.1845	0.184	.6728	0.8418	0.156	.7269	0.7075
0.188	.7496	0.1873	0.174	.6860	0.8455	0.173	.7731	0.7206
0.182	.7656	0.1918	0.186	.6936	0.8477	0.153	.8106	0.7312
0.174	.7739	0.1942	0.184	.7020	0.8500	0.163	.8540	0.7436
0.171	.7822	0.1965	0.198	.7103	0.8524	0.294	737.6454	0.5358
0.178	.7906	0.1989	0.208	.7291	0.8577	0.285	.7301	0.5598
0.167	.8246	0.2086	0.209	.7360	0.8597	0.266	.7388	0.5623
0.176	.8392	0.2127	0.203	.7436	0.8619	0.262	.7954	0.5784
0.174	.8447	0.2143	0.194	.7527	0.8644	0.251	.8405	0.5912
0.286	409.6128	0.4323	0.171	438.6120	0.6626	0.247	.8836	0.6034
0.288	.6253	0.4358	0.177	.6266	0.6668	0.182	738.6413	0.8184
0.292	.6336	0.4382	0.184	.6336	0.6688	0.185	.6830	0.8303
+0.311	.6419	0.4405	+0.181	.6433	0.6715	+0.202	.7264	0.8426

TABLE Ib. (Continued)

V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase
$+0^{\mathrm{m}}.201$	738.7642	0.8533	$+0^{\mathrm{m}}.245$	761.7499	0.3770	$+0^{\mathrm{m}}.260$	793.5815	0.4113
0.213	.8059	0.8652	0.253	.7916	0.3888	0.267	.6242	0.4234
0.222	.8489	0.8774	0.261	.8340	0.4009	0.280	.6787	0.4389
0.237	.8887	0.8887	0.168	766.6183	0.7587	0.283	.7232	0.4515
0.238	.9305	0.9005	0.168	.6576	0.7699	0.288	.7363	0.4552
0.280	742.6404	0.9534	0.165	.7020	0.7825	0.303	.7940	0.4716
0.296	.6720	0.9624	0.175	.7472	0.7953	0.304	.8329	0.4826
0.293	.7239	0.9771	0.179	.7617	0.7994	0.305	795.5828	0.9793
0.310	.7697	0.9901	0.188	.8069	0.8123	0.317	.6155	0.9886
0.306	.8093	0.0014	0.224	769.6201	0.6107	0.316	.6554	0.9999
0.310	.8530	0.0138	0.216	.6593	0.6218	0.314	.6977	0.0119
0.307	.8912	0.0246	0.213	.7003	0.6334	0.294	.7418	0.0244
0.301	.9339	0.0368	0.202	.7413	0.6451	0.292	.7790	0.0350
0.284	.9735	0.0480	0.192	.7829	0.6569	0.301	.8106	0.0439
0.170	750.6407	0.2240	0.181	.8253	0.6689	0.298	818.5734	0.5043
0.165	.6814	0.2356	0.305	772.5978	0.4558	0.300	.6575	0.5282
0.172	.7192	0.2463	0.320	.6385	0.4674	0.281	.6981	0.5397
0.164	.7661	0.2596	0.312	.6798	0.4791	0.267	.7391	0.5514
0.170	.8025	0.2700	0.313	.7228	0.4913	0.279	820.5810	0.0741
0.169	.8463	0.2824	0.306	.7628	0.5026	0.274	.6171	0.0844
0.178	.8883	0.2943	0.293	.8058	0.5148	0.248	.6570	0.0957
0.179	.9300	0.3062	0.206	782.8137	0.3552	0.237	.7001	0.1079
0.182	757.6433	0.2115	0.221	.8470	0.3647	0.230	.7390	0.1190
0.168	.6804	0.2220	0.230	.8901	0.3769	0.214	829.5773	0.6274
0.167	.7221	0.2339	0.283	786.5865	0.4260	0.214	.6162	0.6384
0.154	.7645	0.2459	0.297	.6265	0.4373	0.189	.6554	0.6496
0.170	.8061	0.2577	0.307	.6716	0.4501	0.186	.6974	0.6615
0.160	.8481	0.2696	0.304	.7133	0.4620	0.181	.7270	0.6699
0.174	.8895	0.2814	0.310	.7501	0.4724	0.253	830.6151	0.9219
0.175	.9277	0.2922	0.325	.8032	0.4875	0.260	.6554	0.9334
0.283	760.6221	0.0569	0.316	.8362	0.4968	0.269	.6974	0.9453
0.269	.6683	0.0700	0.164	787.5869	0.7099	0.291	.7411	0.9577
0.259	.7114	0.0822	0.157	.6150	0.7179	0.247	837.5775	0.8980
0.262	.7537	0.0943	0.216	792.6478	0.1463	0.251	.5924	0.9022
0.241	.7961	0.1063	0.203	.6895	0.1581	0.255	.6011	0.9047
0.235	.8374	0.1180	0.193	.7312	0.1699	0.267	.6452	0.9172
0.221	761.6305	0.3431	0.190	.7721	0.1816	0.227	858.6079	0.8667
0.230	.6659	0.3532	0.169	.8135	0.1933	+0.225	.6513	0.8790
+0.231	.7090	0.3654	+0.166	.8513	0.2040			

polation beyond the observing interval then serves to determine the time of minimum light.

From a study of the published spectroscopic conjunctions and selected times of primary eclipse Mannino (1959) has concluded that the orbital period of AO Cas is constant. From spectroscopic data alone Abhyankar (1959), on the other hand, has determined the orbit to be eccentric and the apse to rotate in a period of 70 years. It thus seemed desirable to examine the evidence from the photometric observations only. All published photoelectric light curves were inspected again, and a time of minimum light was derived from each as explained above. These are listed in Table III. An asterisk in the final column denotes a time determined by Koch from the original observations.

It is concluded that within the scatter of the observations primary eclipse can thus far be represented by constant light elements:

$$\text{Pr Min} = 2424002.579 + 3.523487\, E.$$

Short-term departures from these elements are not ruled out. On the average, however, secondary minimum is advanced $0^{\mathrm{d}}.006$ from the half-period point. Only the earliest determination of secondary suggests that it may not be represented by the period given above. Residuals from the new light elements are tabulated in Table III and are represented in Fig. 3.

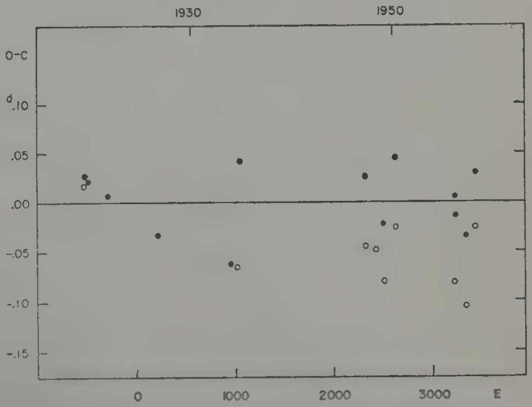


FIG. 3. The residuals given in Table III from the suggested linear light elements. The filled circles refer to primary eclipse and the open ones to secondary eclipse.

Table Ic. Violet Observations of AO Cas.

V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase
$-0^{m}285$	372.7754	0.9709	$-0^{m}285$	381.7903	0.5294	$-0^{m}370$	392.7349	0.6357
0.272	.8303	0.9865	0.306	.8382	0.5430	0.385	.7440	0.6382
0.276	.8629	0.9958	0.289	.8507	0.5466	0.387	.7586	0.6424
0.275	.8782	0.0001	0.297	.8625	0.5499	0.388	.7676	0.6449
0.266	.8921	0.0040	0.327	386.6793	0.9170	0.397	.7759	0.6473
0.413	373.7220	0.2396	0.329	.6994	0.9227	0.402	.7856	0.6500
0.428	.7449	0.2461	0.328	.7112	0.9260	0.402	.7954	0.6528
0.418	.7803	0.2561	0.302	.7231	0.9294	0.400	.8342	0.6638
0.418	.7921	0.2595	0.306	.7418	0.9347	0.405	.8433	0.6664
0.410	.8435	0.2740	0.305	.7536	0.9381	0.380	.8523	0.6690
0.417	.8560	0.2776	0.297	.7668	0.9418	0.394	.8620	0.6717
0.290	374.6922	0.5149	0.301	.7849	0.9470	0.390	.8710	0.6743
0.275	.7234	0.5238	0.286	.7960	0.9501	0.408	.8891	0.6794
0.291	.7561	0.5331	0.283	.8300	0.9598	0.395	.8981	0.6820
0.276	.7672	0.5362	0.270	.8481	0.9649	0.412	.9065	0.6844
0.286	.7811	0.5402	0.275	.8599	0.9683	0.332	393.6579	0.8976
0.324	.8686	0.5650	0.272	.8724	0.9718	0.325	.6704	0.9012
0.331	.8811	0.5685	0.266	.8911	0.9771	0.308	.6787	0.9035
0.330	.8936	0.5721	0.259	.9043	0.9809	0.320	.6870	0.9059
0.331	.9338	0.5835	0.275	.9161	0.9842	0.329	.6954	0.9083
0.338	.9491	0.5878	0.396	387.6689	0.1978	0.328	.7086	0.9120
0.312	376.6811	0.0794	0.427	.7189	0.2120	0.308	.7169	0.9144
0.307	.7006	0.0849	0.416	.7390	0.2177	0.319	.7259	0.9169
0.338	.7339	0.0944	0.422	.7508	0.2211	0.310	.7343	0.9193
0.323	.7464	0.0979	0.396	.7647	0.2250	0.314	.7426	0.9216
0.339	.7582	0.1013	0.410	.7821	0.2300	0.302	.7558	0.9254
0.358	.8395	0.1243	0.424	.7939	0.2333	0.297	.7641	0.9278
0.360	.8534	0.1283	0.438	.8300	0.2436	0.294	.7724	0.9301
0.344	377.6798	0.3628	0.431	.8390	0.2461	0.292	.7808	0.9325
0.352	.7013	0.3689	0.424	.8543	0.2505	0.296	.7891	0.9348
0.337	.7173	0.3735	0.413	.8634	0.2530	0.283	.8350	0.9479
0.345	.7332	0.3780	0.416	.8724	0.2556	0.277	.8433	0.9502
0.332	.7562	0.3845	0.419	.8870	0.2597	0.273	.8516	0.9526
0.323	.7721	0.3890	0.414	.8967	0.2625	0.258	.8600	0.9550
0.324	.7832	0.3922	0.410	.9057	0.2651	0.264	.8683	0.9573
0.298	.8388	0.4080	0.406	.9161	0.2680	0.251	395.6933	0.4753
0.296	.8562	0.4129	0.414	.9370	0.2739	0.244	.7065	0.4790
0.290	.8888	0.4221	0.408	.9460	0.2765	0.235	.7148	0.4814
0.410	378.7715	0.6727	0.262	388.7092	0.4931	0.228	.7232	0.4838
0.406	.7902	0.6780	0.274	.7266	0.4980	0.248	.7315	0.4861
0.419	.8173	0.6857	0.284	.7349	0.5004	0.247	.7447	0.4899
0.414	.8291	0.6890	0.282	.7432	0.5027	0.250	.7530	0.4922
0.426	.8597	0.6977	0.272	.7564	0.5065	0.248	.7683	0.4966
0.434	.8715	0.7010	0.271	.7648	0.5089	0.244	.7766	0.4989
0.445	.8840	0.7046	0.273	390.7064	0.0599	0.246	.7850	0.5013
0.316	379.6833	0.9314	0.297	.7203	0.0610	0.265	397.6614	0.0338
0.297	.7028	0.9370	0.305	.7287	0.0663	0.267	.6697	0.0362
0.294	.7152	0.9405	0.293	.7377	0.0688	0.255	.6781	0.0386
0.296	.7270	0.9438	0.297	.7467	0.0714	0.262	.6864	0.0409
0.285	.7458	0.9492	0.322	.7620	0.0757	0.277	.7024	0.0455
0.281	.7680	0.9555	0.307	.7738	0.0791	0.282	.7107	0.0478
0.286	.7875	0.9610	0.304	.7828	0.0816	0.282	.7197	0.0504
0.273	.8187	0.9699	0.335	.8196	0.0921	0.285	.7281	0.0528
0.270	.8326	0.9738	0.326	.8287	0.0947	0.293	.7364	0.0551
0.273	.8520	0.9793	0.333	.8433	0.0988	0.291	.7496	0.0589
0.266	.8652	0.9831	0.324	.8558	0.1023	0.301	.7579	0.0612
0.276	.8770	0.9864	0.320	.8648	0.1049	0.296	.7662	0.0636
0.282	.8923	0.9908	0.337	.8738	0.1075	0.291	.7746	0.0660
0.288	.9125	0.9965	0.337	391.7454	0.3548	0.294	.7829	0.0683
0.270	.9257	0.0003	0.338	.7613	0.3593	0.315	.8260	0.0805
0.256	.9375	0.0036	0.342	.7717	0.3623	0.312	.8357	0.0833
0.262	.9500	0.0072	0.336	.7808	0.3649	0.317	.8440	0.0857
0.441	380.6924	0.2179	0.344	.7905	0.3676	0.318	.8524	0.0880
0.414	.7139	0.2240	0.330	.8301	0.3789	0.299	.8607	0.0904
0.418	.7257	0.2273	0.321	.8398	0.3816	0.380	398.6558	0.3161
0.405	.7375	0.2307	0.315	.8502	0.3846	0.392	.6683	0.3196
0.433	.7562	0.2360	0.318	.8606	0.3875	0.380	.6760	0.3218
0.420	.7680	0.2393	0.319	.8710	0.3905	0.377	.6843	0.3241
0.433	.7826	0.2435	0.309	.8814	0.3934	0.375	.6926	0.3265
0.430	.8375	0.2590	0.308	.9044	0.3999	0.378	.7051	0.3300
0.415	.8493	0.2624	0.300	.9155	0.4031	0.377	.7135	0.3324
0.408	.8639	0.2665	0.385	392.7099	0.6286	0.370	.7218	0.3348
−0.273	381.7716	0.5241	−0.377	.7245	0.6327	−0.372	.7301	0.3371

TABLE Ic. (Continued)

V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase
−0^{m}376	398.7385	0.3395	−0^{m}398	410.6322	0.7216	−0^{m}421	438.6676	0.6784
0.363	.7690	0.3482	0.401	.6405	0.7240	0.405	.7072	0.6897
0.367	.7774	0.3506	0.405	.6489	0.7263	0.436	.7252	0.6948
0.361	.7855	0.3529	0.408	.6572	0.7287	0.418	.7342	0.6973
0.353	.7954	0.3557	0.418	.6704	0.7324	0.411	.7474	0.7011
0.352	.8301	0.3655	0.416	.6787	0.7348	0.295	439.6154	0.9474
0.325	.8433	0.3693	0.407	.6871	0.7372	0.290	.6293	0.9514
0.329	.8531	0.3720	0.411	.6954	0.7395	0.291	.6363	0.9534
0.330	.8614	0.3744	0.410	.7037	0.7419	0.297	.6432	0.9553
0.322	.8697	0.3768	0.415	.7169	0.7456	0.295	.6502	0.9573
0.322	.8781	0.3791	0.421	.7252	0.7480	0.289	.6592	0.9599
0.455	403.6336	0.7288	0.422	.7336	0.7504	0.274	.6745	0.9642
0.412	.6468	0.7326	0.405	.7419	0.7527	0.274	.6821	0.9664
0.417	.6552	0.7350	0.405	.7558	0.7567	0.282	.6904	0.9687
0.422	.6635	0.7373	0.412	.7641	0.7590	0.285	.6974	0.9707
0.426	.6718	0.7397	0.416	.7725	0.7614	0.272	.7071	0.9735
0.427	.6843	0.7432	0.410	.8010	0.7695	0.256	.7342	0.9811
0.425	.6926	0.7456	0.410	.8093	0.7719	0.258	.7453	0.9843
0.424	.7010	0.7477	0.405	.8225	0.7756	0.258	.7530	0.9865
0.437	.7093	0.7503	0.400	.8308	0.7780	0.268	.7606	0.9886
0.434	.7176	0.7527	0.407	.8391	0.7803	0.253	.7682	0.9908
0.432	.7302	0.7562	0.395	.8475	0.7827	0.427	440.6066	0.2287
0.427	.7385	0.7586	0.289	411.6204	0.0020	0.427	.6208	0.2328
0.427	.7468	0.7610	0.278	.6343	0.0060	0.425	.6295	0.2352
0.424	.7552	0.7633	0.265	.6426	0.0083	0.431	.6388	0.2379
0.424	.7635	0.7657	0.277	.6523	0.0111	0.426	.6486	0.2406
0.425	.8024	0.7767	0.271	.6607	0.0135	0.418	.6642	0.2451
0.426	.8107	0.7791	0.266	.6746	0.0174	0.430	.6736	0.2477
0.418	.8190	0.7814	0.273	.6829	0.0198	0.412	.6836	0.2506
0.403	.8274	0.7838	0.274	.6912	0.0221	0.425	449.5976	0.7805
0.427	.8364	0.7864	0.272	.7030	0.0255	0.428	.6129	0.7848
0.417	.8496	0.7901	0.278	.7114	0.0279	0.423	.6202	0.7869
0.425	.8579	0.7925	0.280	.7197	0.0302	0.425	.6372	0.7917
0.409	.8663	0.7949	0.278	.7357	0.0348	0.426	.6441	0.7937
0.413	.8746	0.7972	0.294	.7440	0.0371	0.363	452.6124	0.6361
0.326	407.7461	0.8960	0.279	.7537	0.0399	0.386	.6273	0.6404
0.323	.7586	0.8995	0.285	.7620	0.0422	0.399	.6343	0.6424
0.317	.7670	0.9019	0.289	.7711	0.0448	0.388	.6412	0.6443
0.321	.7753	0.9043	0.289	.7850	0.0488	0.369	.6488	0.6465
0.318	.7836	0.9066	0.295	.8162	0.0576	0.386	458.5832	0.3307
0.315	.7996	0.9112	0.302	.8246	0.0600	0.380	.5957	0.3343
0.314	.8079	0.9135	0.288	.8329	0.0624	0.383	.6027	0.3363
0.301	.8176	0.9163	0.301	.8412	0.0647	0.381	.6096	0.3382
0.303	.8274	0.9191	0.294	413.6460	0.5769	0.376	.6186	0.3408
0.303	.8357	0.9214	0.327	.6627	0.5817	0.289	476.5869	0.4404
0.347	408.6190	0.1502	0.328	.6710	0.5840	0.286	.5994	0.4440
0.362	.6322	0.1540	0.332	.6794	0.5864	0.282	.6064	0.4460
0.374	.6406	0.1564	0.347	.6912	0.5898	0.281	.6133	0.4479
0.361	.6489	0.1587	0.334	.7120	0.5957	0.284	.6216	0.4503
0.367	.6572	0.1611	0.354	.7981	0.6201	0.275	.6348	0.4540
0.365	.6697	0.1646	0.354	.8162	0.6252	0.281	.6418	0.4560
0.366	.6780	0.1670	0.357	.8245	0.6276	0.322	732.6758	0.1254
0.355	.6864	0.1694	0.358	.8363	0.6309	0.355	.7431	0.1445
0.368	.6947	0.1717	0.392	428.6221	0.8274	0.366	.7897	0.1577
0.379	.7030	0.1741	0.398	.6346	0.8309	0.372	.8320	0.1697
0.391	.7156	0.1776	0.398	.6443	0.8337	0.377	.8563	0.1766
0.375	.7239	0.1800	0.400	.6554	0.8368	0.355	734.6217	0.6776
0.380	.7322	0.1824	0.399	.6638	0.8392	0.398	.6751	0.6928
0.383	.7406	0.1847	0.383	.6749	0.8424	0.418	.7276	0.7077
0.383	.7503	0.1875	0.404	.6867	0.8457	0.421	.7738	0.7208
0.382	.7662	0.1920	0.387	.6943	0.8479	0.433	.8113	0.7314
0.386	.7746	0.1944	0.381	.7041	0.8506	0.414	.8547	0.7438
0.397	.7829	0.1967	0.366	.7110	0.8526	0.256	737.6458	0.5359
0.396	.7912	0.1991	0.374	.7298	0.8579	0.278	.7304	0.5599
0.403	.8253	0.2088	0.377	.7367	0.8599	0.290	.7392	0.5624
0.391	.8398	0.2129	0.363	.7457	0.8625	0.310	.7958	0.5785
0.387	.8454	0.2145	0.369	.7548	0.8650	0.322	.8409	0.5913
0.278	409.6135	0.4325	0.400	438.6127	0.6628	0.327	.8840	0.6035
0.271	.6260	0.4360	0.399	.6273	0.6670	0.375	738.6417	0.8185
0.262	.6343	0.4384	0.395	.6342	0.6689	0.375	.6833	0.8304
0.235	.6426	0.4407	0.390	.6440	0.6717	0.380	.7267	0.8427
0.285	.6503	0.4429	0.398	.6523	0.6741	0.379	.7646	0.8534
−0.394	410.6197	0.7180	−0.409	.6592	0.6760	−0.364	.8062	0.8652

TABLE Ic. (Continued)

V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase	V−C	JD ⊙ 2435000+	Phase
$-0^{\mathrm{m}}354$	738.8493	0.8775	$-0^{\mathrm{m}}321$	761.7923	0.3890	$-0^{\mathrm{m}}325$	793.5822	0.4115
0.335	.8890	0.8887	0.312	.8346	0.4010	0.294	.6249	0.4236
0.331	.9309	0.9006	0.423	766.6190	0.7589	0.302	.6794	0.4391
9.263	742.6407	0.9535	0.423	.6597	0.7705	0.289	.7238	0.4517
0.256	.6723	0.9625	0.425	.7027	0.7827	0.281	.7370	0.4554
0.260	.7246	0.9773	0.408	.7479	0.7955	0.270	.7947	0.4718
0.264	.7704	0.9903	0.416	.7624	0.7996	0.258	.8336	0.4828
0.255	.8100	0.0016	0.385	.8076	0.8124	0.261	795.5835	0.9795
0.247	.8537	0.0140	0.342	769.6208	0.6109	0.250	.6161	0.9887
0.265	.8919	0.0248	0.355	.6600	0.6220	0.265	.6561	0.0001
0.254	.9346	0.0370	0.372	.7010	0.6336	0.272	.6984	0.0121
0.273	.9742	0.0482	0.379	.7420	0.6453	0.271	.7425	0.0246
0.404	750.6414	0.2242	0.387	.7836	0.6571	0.277	.7797	0.0352
0.403	.6821	0.2358	0.384	.8260	0.6691	0.267	.8113	0.0441
0.398	.7199	0.2465	0.242	772.5985	0.4560	0.282	818.5741	0.5045
0.408	.7668	0.2598	0.251	.6392	0.4676	0.269	.6568	0.5280
0.401	.8032	0.2702	0.262	.6805	0.4793	0.292	.6988	0.5399
0.393	.8470	0.2826	0.262	.7235	0.4915	0.293	.7398	0.5516
0.395	.8890	0.2945	0.264	.7635	0.5028	0.305	820.5817	0.0743
0.389	.9307	0.3063	0.278	.8065	0.5150	0.308	.6178	0.0846
0.393	757.6440	0.2117	0.365	782.8143	0.3554	0.329	.6577	0.0959
0.414	.6811	0.2222	0.355	.8477	0.3649	0.341	.7008	0.1081
0.415	.7228	0.2341	0.320	.8908	0.3771	0.338	.7397	0.1192
0.409	.7652	0.2461	0.291	786.5872	0.4262	0.364	829.5780	0.6276
0.400	.8068	0.2579	0.280	.6272	0.4375	0.355	.6169	0.6386
0.410	.8488	0.2698	0.259	.6723	0.4503	0.375	.6561	0.6498
0.394	.8902	0.2816	0.265	.7140	0.4622	0.368	.6981	0.6617
0.386	.9284	0.2924	0.254	.7508	0.4726	0.378	.7277	0.6701
0.293	760.6228	0.0571	0.230	.8039	0.4877	0.318	830.6158	0.9221
0.307	.6690	0.0702	0.235	.8369	0.4970	0.308	.6561	0.9336
0.313	.7121	0.0824	0.397	787.5876	0.7101	0.291	.6981	0.9455
0.318	.7544	0.0945	0.416	.6157	0.7181	0.261	.7418	0.9579
0.336	.7968	0.1065	0.369	792.6485	0.1465	0.333	837.5782	0.8982
0.347	.8381	0.1182	0.375	.6902	0.1583	0.322	.5931	0.9024
0.356	761.6312	0.3433	0.401	.7319	0.1701	0.322	.6018	0.9049
0.345	.6666	0.3533	0.388	.7728	0.1818	0.299	.6459	0.9174
0.343	.7096	0.3656	0.399	.8142	0.1935	0.348	858.6086	0.8669
−0.330	.7505	0.3772	−0.390	.8520	0.2042	−0.328	.6520	0.8792

If primary eclipse is to occur at $0^{\mathrm{p}}0$ for the Tucson observations, different light elements are required for each observing interval. These follow:

$$\text{Pr Min (I)}=2432191.189+3.523465\,E,$$

$$\text{Pr Min (II)}=2432191.189+3.523440\,E,$$

$$\text{Pr Min (III)}=2432191.189+3.523428\,E.$$

While these period changes are small, they may very well be real. Moreover, they can be expected in a system such as AO Cas since Wood (1948) has shown the bright component to be near the limit of dynamical stability.

4. VARIATIONS OUTSIDE ECLIPSE, TUCSON OBSERVATIONS

According to the method given by Russell anc Merrill (1952) the light variation outside eclipse wa represented by

$$\mathcal{L}=A_0+A_1\cos\theta+A_2\cos2\theta+A_3\cos3\theta+A_4\cos4\theta+$$
$$+B_1\sin\theta+B_2\sin2\theta+B_3\sin3\theta+B_4\sin4\theta$$

The light curves from Intervals I and III were analyzec individually. The curves for the second interval are no covered well enough to warrant discussion. The result of the analyses are given in Table IV in units of 0.0001 This table also lists for a single observation the probabl

TABLE II. Magnitudes and colors of comparison stars and AO Cas.

Star	V	n	$(B-V)$	n	$(U-B)$	n
BD +47°50	$+5^{\mathrm{m}}89\pm0^{\mathrm{m}}01$	13	$-0^{\mathrm{m}}09\pm0^{\mathrm{m}}01$	8	$-0^{\mathrm{m}}44\pm0^{\mathrm{m}}02$	8
BD +51°62	5.62±0.02	3	−0.12±0.01	3	−0.59±0.02	3
BD +51°62	5.56	—	−0.12	—	−0.59	—
AO Cas	(6.24–6.09)	—	−0.13±0.01	6	−1.00±0.02	6
AO Cas	+5.9	1	+0.03	1	−1.01	1

residual in magnitudes from the computed variation. The plots of $\mathfrak{L}$, transformed to magnitude scale, against the yellow and violet observations are shown in Figs. 1 and 2. The fact that several of the computed variations run fainter than the observations at the eclipses indicates that at least some of the complication effects are not visible within the eclipses.

For the first interval the observations do not cover the light curve uniformly so that the computed probable residuals are not realistic determinations. Values similar to those found for the third interval are more appropriate.

In the entries of Table IV several regularities are apparent: (1) The coefficient of $\cos\theta$, conventionally understood to be due to differential reflection within the system, is always either zero or negative. It is thus always of the expected algebraic sign. The negative values are satisfactorily small as they should be for stars of nearly equal surface temperatures; (2) The $\cos 2\theta$ term shows only small changes so that it substantiates Wood's (1948) conclusion that it may be assumed constant; (3) Most of the remaining terms are small. It might even be permissible to ignore harmonics higher than the second and consider the representations of the variations adequate. In view of the small but recognizable values of A_1 uncovered by the analyses, it is considered desirable to retain as real those coefficients which are not algebraically smaller than ±0.0010; (4) No apparent wavelength dependence exists for any of the coefficients.

5. VARIATIONS OUTSIDE ECLIPSE, OTHER LIGHT CURVES

Because the Tucson observations showed small variations judged to be real in harmonics higher than 2θ, it was decided to return to the light curves previously published and inspect them for similar terms. This was deemed worthwhile for Guthnick's curve of 1919–20, Wood's curve of 1946–47, and an unpublished ultraviolet curve observed by Wood in 1949–50. These results are collected in Table V for the same entries as those of Table IV. In general, the regularities which were uncovered in the Tucson observations also appear in these results. For Wood's 1946–47 curve this analysis does not disagree with the variation which he published.

Table III. Times of minimum light, AO Cas.

JD ⊙ 2400000+	E	$O-C$	Reference
22212.675	−508	$+0^{\mathrm{d}}027$	Guthnick (1920)
22300.757	−483	+0.022	Güssow (1930)
22348.320	−469.5	+0.018	Guthnick (1920)*
22987.822	−288	+0.007	Güssow (1930)
24770.667	+218	−0.032	Güssow (1930)
27399.159	+964	−0.061	Bennett (1939)
27418.536	+969.5	−0.064	Bennett (1939)*
27751.610	+1064	+0.041	Bennett (1939)*
32191.189	+2324	+0.026	Wood (1948)
32192.881	+2324.5	−0.044	Wood (1948)*
32527.608	+2419.5	−0.048	Wood (1952)
32821.845	+2503	−0.022	Hiltner (1949)
32844.690	+2509.5	−0.080	Hiltner (1949)
33255.301	+2626	+0.045	Wood (unpublished)*
33256.993	+2626.5	−0.025	Wood (unpublished)*
35372.878	+3227	+0.006	Koch
35374.552	+3227.5	−0.081	Koch
35411.617	+3238	−0.013	Koch
35786.776	+3344.5	−0.105	Koch
35795.656	+3347	−0.034	Koch
36096.923	+3432.5	−0.025	Abhyankar (1959)*
36151.592	+3448	+0.030	Abhyankar (1959)*

6. DISCUSSION

Before 1955 nine photoelectric light curves of AO Cas had been observed. The Tucson observations have added nine more, and Abhyankar, three, and an examination of some of the previous conclusions about the system is in order.

From a study of his yellow curve Hiltner (1949) calls attention to the similarities between secondary minimum in AO Cas and the minimum of eclipse of the Wolf-Rayet component of CQ Cep. Without exception

Table IV. Outside-eclipse variations, Tucson observations.

Int.	A_0	A_1	A_2	A_3	A_4	B_1	B_2	B_3	B_4	Residual
I, y	+9496	−19	−660	0	−19	−44	+107	−32	+12	$\pm0^{\mathrm{m}}002$
I, b	9380	0	619	+16	0	57	85	−14	+14	0.002
I, u	9557	0	681	0	−18	77	98	−24	0	0.002
III, y	9400	−8	628	−13	−7	54	37	+4	+17	0.005
III, b	9304	0	588	0	−8	43	25	−21	+8	0.005
III, u	+9490	0	−639	−40	−15	−58	+22	0	−7	±0.008

Table V. Outside-eclipse variations, old observations.

Light curve	A_0	A_1	A_2	A_3	A_4	B_1	B_2	B_3	B_4	Residual
Guthnick (1919–20)	+9181	−14	−658	+28	−48	−110	+50	+46	−19	$\pm0^{\mathrm{m}}006$
Wood (1946–47)	9264	0	583	−90	8	72	82	+19	+12	0.013
Wood (1949–50)	+9634	0	−498	−104	−25	−72	+138	−12	−9	±0.013

the Tucson observations confirm his finding that secondary eclipse is more sharply pointed than the primary.

From the observations of Hiltner (1949) and Abhyankar and the Tucson light curves it appears that color variations of AO Cas do not exceed $0^{m}.01$. Thus, the observed light variations due to local gravity changes across the surfaces of the stars must be quite small. The orbital inclination is low so that this may be only an effect of projection. It could also happen that the effect of gravity darkening is presently compensated at every position in the revolution. If this is actually the case, the system could change so that the gravity effect could be observed.

In general, the conclusions of Wood (1948) are confirmed by the new light curves. The variations of the terms in $\sin\theta$ and $\sin 2\theta$ are well established, and the presence of higher harmonics has been at least provisionally indicated. Attempts at rectifying the new observations show the rectified depths of eclipse to be similar to those used by Wood for his limiting solution for the elements of the system. Accordingly, a new solution is not attempted.

For all the light curves studied in Secs. 4 and 5 the coefficient of $\sin\theta$ is negative so that the brightest point of the light curve occurs at the maximum following secondary. There has been only one indication of the contrary situation when the maximum following primary eclipse is brightest. This occurred in the fragmentary light curve of the second interval of the Tucson observations. It was at this time that an abrupt change in shape of the light curve was detected.

7. SUMMARY

Study of numerous observations of AO Cas has permitted a review of previous conclusions about the system. A variety of problems remain, and among them we can enumerate two which merit attention: (1) The limits of variation of the terms in $\cos\theta$, $\cos 2\theta$, $\sin\theta$, and $\sin 2\theta$ should be determined. (2) The constancy of the proposed period should be tested. It is felt that photoelectric photometry with interference filters can contribute to the answers to these problems and others but that a significant advance awaits simultaneous photometric and spectroscopic observation.

8. ACKNOWLEDGMENTS

This paper, in a more extensive form, was received by the University of Pennsylvania in partial fulfillment of the requirements for the Doctor's degree in astronomy. I would like to acknowledge my gratitude to Professors E. F. Carpenter and W. S. Fitch of the Steward Observatory for permitting me the use of the Observatory's facilities. At the University of Pennsylvania Drs. F. B. Wood, J. E. Merrill, L. Binnendijk, W. Blitzstein, W. M. Protheroe, and K. T. Menon kindly offered assistance and advice. Financial assistance for this research was received from the University of Pennsylvania (Harrison and Pawling Fellowships) and from the University of Arizona (Steward Observatory Fellowship). This aid is gratefully acknowledged.

REFERENCES

Abhyankar, K. D. 1959, *Astrophys. J. Suppl. Ser.* **4**, 187.
Bennett, A. L. 1939, *Astron. J.* **47**, 104.
Güssow, M. 1930, *Astron. Nachr.* **237**, 327.
Guthnick, P. 1920, *Astron. Nachr.* **211**, 391.
Hiltner, W. A. 1949, *Astrophys. J.* **110**, 443.
——. 1956, *Astrophys. J. Suppl. Ser.* **2**, 395.
Hiltner, W. A. and Johnson, H. L. 1956, *Astrophys. J.* **124**, 367.
Mannino, G. 1959, *Mem. della Soc. Astr. Ital.* **30**, 19.
Russell, H. N. and Merrill, J. E. 1952, *Princeton Contr.* **26**, 53.
Wood, F. B. 1948, *Astrophys. J.* **108**, 28.
——. 1952, *Astron. J.* **56**, 53.

THE ASTRONOMICAL JOURNAL VOLUME 65, NUMBER 3 APRIL, 1960

Photoelectric Photometry of AS Eridani

ROBERT H. KOCH*
Flower and Cook Observatory, Philadelphia, Pennsylvania

(Received December 18, 1959)

Photoelectric light curves of AS Eri have been obtained in yellow and blue light and have been analyzed for the orbital elements. Additional observations place the binary on the B,V system. The binary is shown to be probably detached and the hot component to be close to the initial main sequence.

1. INTRODUCTION

HOFFMEISTER (1934) discovered the light variation of AS Eri (BD $-3°570$, 271.1934, HD 21985, P 2575, 1950.0: $\alpha=3^h\ 30^m.1$, $\delta=-3°\ 28'$). He gives the photographic amplitudes as $0^m.6$ in primary eclipse and $0^m.1$ in secondary and classifies the binary as Algol type. A number of heliocentric times of minimum light have been estimated visually and observed photoelectrically. Also, proper motions have been given by Kopal (1942). From estimates of the Harvard patrol plates Gaposchkin (1946, 1953) has published a mean light curve, and from McDonald spectrograms he has given a radial velocity curve. The solutions from these observations determine primary eclipse to be annular with an A0 main sequence star being eclipsed by an F5 dwarf. Using Gaposchkin's solution, Huang and Struve (1956) have demonstrated the possibility that the hot star may at times be surrounded by an emitting ring of matter ejected from the cool star.

2. THE OBSERVATIONS

From 1955 to 1957 a total of 586 yellow and 572 blue observations were obtained with the photoelectric photometer on the 36-in. Steward reflector in Tucson. All observations were corrected for differential extinction using the mean coefficients, $0^m.220$ in yellow and $0^m.353$ in blue. All observations are listed in Table I, and those for the second season are shown in Figs. 1 and 2.

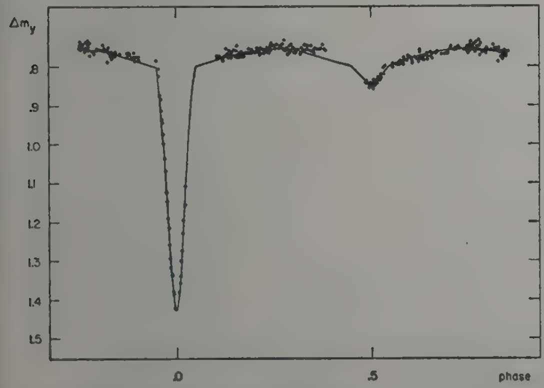


FIG. 1. Yellow observations of AS Eri during 1956–1957 observing season. The light curve is given in Table V.

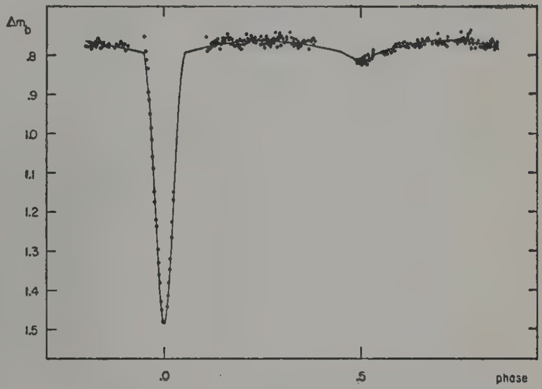


FIG. 2. Blue observations of AS Eri during 1956–1957 observing season. The light curve is given in Table V.

The comparison star was BD $-3°576$ and at least once a night was checked against BD $-3°565$. Within each season there is no evidence of variation of the comparison star. The mean seasonal magnitude differences of the check and comparison stars agree within the errors of observation.

On two nights the comparison and check stars were observed in both yellow and blue light along with κ Cet and BD $+24°547$. With these latter as standards it is possible to transform to the B,V system. Table II lists the results giving probable errors and the number of observations.

No violet observations were taken. In the Henry Draper Catalogue the comparison star is given as A2 and the check star as F0. According to Johnson (1955), unreddened stars of these types may be assumed to have the following average colors: for A2 (IV–V), $+0^m.04$ and for F0 (IV–V), $+0^m.33$. It appears then that the comparison stars are reddened by $0^m.07$ to $0^m.10$. Even if the spectral classifications are too early by one- or two-tenths of a class, there is still an indication of a slight amount of reddening.

Hubble (1934) has discussed the distribution of extra-galactic nebulae in several fields surrounding AS Eri ($l=156°$, $b=-43°$). According to his analysis the distribution of galaxies varies from moderate deficiencies to moderate excesses. On the print of the red plate of the National Geographic-Palomar Sky Atlas there is

---

* Now at Amherst College Observatory, Amherst, Massachusetts.

Table Ia. Yellow observations of AS Eridani.

V-C	JD ⊙ 2435000+	Phase
+0$^{\rm m}$757	426.7751	0.7042
0.759	.7799	0.7060
0.755	.7876	0.7089
0.753	.8008	0.7139
0.754	.8056	0.7157
0.759	.8105	0.7175
0.748	.8154	0.7194
0.755	.8251	0.7230
0.751	.8272	0.7238
0.745	.8362	0.7272
0.757	.8438	0.7300
0.775	427.7119	0.0559
0.778	.7230	0.0600
0.776	.7355	0.0647
0.781	.7938	0.0866
0.782	.7987	0.0884
0.768	.8640	0.1129
0.764	.8778	0.1181
0.774	431.8670	0.6155
0.775	432.6834	0.9219
0.776	.6854	0.9227
0.776	.6969	0.9270
0.793	.7028	0.9292
0.790	.7090	0.9316
0.775	.7153	0.9339
0.792	.7250	0.9376
0.784	.7341	0.9410
0.780	.7403	0.9433
0.786	.7466	0.9457
0.804	.7528	0.9480
0.802	.7590	0.9503
0.893	.7840	0.9597
0.896	.7903	0.9621
1.137	.8313	0.9775
1.172	.8389	0.9803
1.243	.8479	0.9837
1.304	.8570	0.9871
1.355	.8632	0.9894
1.355	.8695	0.9918
1.416	.8764	0.9944
1.429	.8862	0.9981
1.432	.8931	0.0007
1.431	.8993	0.0030
1.411	.9063	0.0056
1.349	.9181	0.0100
1.309	.9271	0.0134
0.799	436.7082	0.4327
0.786	.7221	0.4380
0.774	.7284	0.4403
0.775	.7374	0.4436
0.786	.7444	0.4463
0.803	.7534	0.4496
0.795	.7603	0.4522
0.792	452.7084	0.4384
0.773	.7188	0.4423
0.774	.7292	0.4462
0.785	.7362	0.4488
0.776	.7428	0.4513
0.781	.7515	0.4546
0.790	.7598	0.4577
0.784	.7688	0.4611
0.783	.7751	0.4634
0.802	.7834	0.4666
0.795	.7897	0.4689
0.813	.7987	0.4723
0.816	.8084	0.4759
0.822	.8154	0.4786
0.813	.8230	0.4814
0.814	.8313	0.4845
0.745	453.7688	0.8364
0.755	.7771	0.8396
0.762	.7848	0.8424
+0.787	456.6704	0.9255
+0$^{\rm m}$784	456.6790	0.9288
0.785	.6846	0.9309
0.782	.6909	0.9332
0.779	.6971	0.9356
0.789	.7034	0.9379
0.776	.7124	0.9413
0.787	.7207	0.9444
0.790	.7270	0.9468
0.789	.7332	0.9491
0.820	.7395	0.9515
0.828	.7485	0.9549
0.924	.7693	0.9627
0.925	.7721	0.9637
0.962	.7784	0.9661
0.999	.7846	0.9684
1.041	.7909	0.9708
1.091	.7992	0.9739
1.125	.8054	0.9762
1.165	.8117	0.9786
1.191	.8138	0.9794
1.239	.8207	0.9820
1.289	.8270	0.9843
1.316	.8332	0.9867
1.346	.8395	0.9890
1.380	.8457	0.9913
1.412	.8520	0.9937
1.412	.8582	0.9960
1.431	.8645	0.9984
1.423	.8707	0.0007
1.415	.8770	0.0031
1.393	.8846	0.0059
1.374	.8895	0.0078
0.748	458.6748	0.6779
0.749	.6866	0.6824
0.752	.6928	0.6847
0.748	.6991	0.6870
0.751	.7060	0.6896
0.738	.7123	0.6920
0.743	.7192	0.6946
0.746	.7276	0.6977
0.743	.7338	0.7001
0.749	.7401	0.7024
0.743	.7463	0.7048
0.744	.7533	0.7074
0.753	.7644	0.7116
0.807	460.7934	0.4731
0.810	.8038	0.4770
0.809	.8101	0.4794
0.751	473.6731	0.3076
0.754	.6815	0.3107
0.736	.6908	0.3142
0.763	.6988	0.3172
0.976	491.5922	0.0336
0.913	.6002	0.0366
0.901	.6068	0.0391
0.857	.6130	0.0414
0.836	.6193	0.0438
0.813	.6252	0.0460
0.797	.6314	0.0483
0.798	.6401	0.0516
0.780	.6460	0.0538
0.790	.6519	0.0560
0.797	.6578	0.0582
0.782	.6634	0.0603
0.792	.6693	0.0625
0.793	.6752	0.0647
0.787	.6845	0.0682
0.789	.6911	0.0707
0.772	.6988	0.0736
0.789	.7054	0.0761
0.779	.7130	0.0789
0.768	.7196	0.0814
+0.769	.7300	0.0853
+0$^{\rm m}$790	491.7387	0.0886
0.819	507.6039	0.0437
0.816	.6147	0.0477
0.776	.6209	0.0500
0.791	.6446	0.0589
0.787	.6532	0.0622
0.782	.6595	0.0645
0.777	.6661	0.0670
0.794	.6741	0.0700
0.787	.6817	0.0729
0.781	508.5962	0.4161
0.774	.6063	0.4199
0.779	.6177	0.4242
0.783	.6230	0.4262
0.788	.6288	0.4284
0.775	.6351	0.4307
0.773	.6406	0.4328
0.786	.6479	0.4355
0.791	.6538	0.4377
0.785	.6608	0.4404
0.787	.6695	0.4436
0.777	.6754	0.4458
0.794	.6803	0.4477
0.779	.6872	0.4503
0.773	.6931	0.4525
0.791	.6990	0.4547
0.784	.7059	0.4573
0.798	.7160	0.4611
0.791	.7222	0.4634
0.752	513.6141	0.2996
0.738	.6231	0.3030
0.746	.6290	0.3052
0.748	.6349	0.3074
0.740	.6408	0.3096
0.744	.6484	0.3125
0.746	.6568	0.3156
0.756	514.5932	0.6671
0.757	.6026	0.6706
0.749	.6088	0.6730
0.753	.6144	0.6751
0.747	.6199	0.6771
0.758	.6259	0.6794
0.732	.6324	0.6818
0.762	517.6012	0.7962
0.751	.6099	0.7994
0.759	.6158	0.8016
0.763	518.5970	0.1700
0.752	.6057	0.1732
0.745	.6116	0.1754
0.940	733.8486	0.9655
0.753	755.8923	0.2397
0.755	.8986	0.2420
0.748	.9052	0.2445
0.740	.9097	0.2462
0.747	.9166	0.2488
0.748	.9208	0.2504
0.742	.9264	0.2525
0.744	.9305	0.2540
0.751	.9364	0.2562
0.743	.9406	0.2578
0.748	.9465	0.2600
0.754	.9507	0.2616
0.746	.9573	0.2641
0.751	.9614	0.2656
0.746	.9670	0.2677
0.747	.9711	0.2692
0.740	.9767	0.2713
0.743	.9809	0.2729
0.741	.9885	0.2758
0.732	.9927	0.2773
0.794	756.8059	0.5826
0.787	.8156	0.5862
+0.782	.8198	0.5878

TABLE Ia.—*Continued.*

V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase
+0$^{\mathrm{m}}$.779	756.8268	0.5904	+0$^{\mathrm{m}}$.746	760.9964	0.1555	+1$^{\mathrm{m}}$.424	765.9056	0.9982
0.783	.8313	0.5921	0.840	761.8825	0.4881	1.380	.9316	0.0080
0.776	.8372	0.5943	0.849	.8926	0.4919	1.359	.9358	0.0095
0.771	.8413	0.5959	0.840	.8968	0.4935	1.340	.9407	0.0114
0.778	.8472	0.5981	0.853	.9023	0.4955	1.301	.9473	0.0139
0.787	.8514	0.5997	0.844	.9065	0.4971	1.275	.9518	0.0155
0.776	.8580	0.6021	0.840	.9120	0.4992	1.227	.9577	0.0178
0.783	.8622	0.6037	0.844	.9162	0.5008	1.197	.9622	0.0194
0.782	.8677	0.6058	0.845	.9204	0.5023	1.156	.9684	0.0218
0.786	.8719	0.6074	0.840	.9266	0.5047	1.109	.9743	0.0240
0.774	.8788	0.6100	0.834	.9318	0.5066	0.838	769.8766	0.4887
0.780	.8830	0.6115	0.842	.9360	0.5082	0.847	.8852	0.4920
0.783	.8886	0.6136	0.843	.9426	0.5107	0.841	.8908	0.4941
0.779	.8927	0.6152	0.835	.9468	0.5122	0.844	.8977	0.4967
0.780	.8983	0.6173	0.845	.9513	0.5139	0.846	.9019	0.4982
0.783	.9024	0.6188	0.841	.9568	0.5160	0.843	.9075	0.5003
0.789	.9080	0.6209	0.834	.9613	0.5177	0.850	.9127	0.5023
0.779	.9122	0.6225	0.826	.9652	0.5192	0.854	.9189	0.5046
0.755	.9184	0.6248	0.835	.9718	0.5216	0.844	.9231	0.5062
0.761	.9226	0.6264	0.822	.9759	0.5232	0.841	.9297	0.5087
0.772	.9285	0.6286	0.822	.9818	0.5254	0.849	.9338	0.5102
0.781	.9330	0.6303	0.828	.9884	0.5279	0.841	.9398	0.5125
0.778	.9389	0.6325	0.801	.9968	0.5310	0.833	.9436	0.5139
0.783	.9431	0.6341	0.752	762.7628	0.8185	0.840	.9523	0.5172
0.771	.9493	0.6364	0.764	.7826	0.8260	0.837	.9571	0.5190
0.770	.9535	0.6380	0.751	.7871	0.8277	0.836	.9637	0.5214
0.767	.9590	0.6401	0.761	.8298	0.8437	0.832	.9686	0.5233
0.768	.9632	0.6416	0.763	.8336	0.8451	0.819	.9752	0.5258
0.771	.9688	0.6437	0.761	.8572	0.8540	0.822	.9814	0.5281
0.781	.9729	0.6453	0.768	.8613	0.8555	0.816	.9880	0.5306
0.773	.9813	0.6484	0.770	.8673	0.8578	0.800	.9922	0.5321
0.780	.9854	0.6500	0.766	.8711	0.8592	0.767	783.7084	0.6806
0.773	.9910	0.6521	0.763	.8746	0.8605	0.767	.7345	0.6904
0.764	.9952	0.6536	0.767	.9145	0.8755	0.759	.7407	0.6927
0.768	757.0014	0.6560	0.774	.9187	0.8771	0.756	.7466	0.6949
0.777	.0056	0.6575	0.764	.9249	0.8794	0.761	.7511	0.6966
0.750	759.9036	0.7453	0.769	.9291	0.8810	0.763	.7546	0.6979
0.765	.9130	0.7489	0.777	.9343	0.8829	0.757	.7605	0.7001
0.751	.9172	0.7504	0.780	.9399	0.8850	0.756	.7643	0.7015
0.746	.9234	0.7528	0.774	.9451	0.8870	0.759	.7713	0.7042
0.736	.9276	0.7543	0.778	.9492	0.8885	0.750	.7747	0.7054
0.738	.9338	0.7567	0.775	.9548	0.8906	0.754	.7810	0.7078
0.752	.9380	0.7583	0.763	.9590	0.8922	0.754	.7845	0.7091
0.759	.9439	0.7605	0.776	.9631	0.8937	0.759	.7907	0.7114
0.741	.9481	0.7620	0.787	.9690	0.8959	0.757	.7945	0.7129
0.752	.9543	0.7644	0.784	.9732	0.8975	0.759	.8008	0.7152
0.751	.9588	0.7661	0.777	.9774	0.8991	0.754	.8042	0.7165
0.739	.9658	0.7687	0.779	.9833	0.9013	0.758	.8105	0.7189
0.750	.9699	0.7702	0.778	.9874	0.9028	0.750	.8143	0.7203
0.750	.9769	0.7729	0.778	.9919	0.9045	0.750	.8202	0.7225
0.733	.9824	0.7749	0.786	.9968	0.9064	0.756	.8240	0.7239
0.743	.9890	0.7774	0.782	763.0010	0.9079	0.755	.8643	0.7391
0.740	.9953	0.7798	0.807	765.7865	0.9535	0.754	.8685	0.7406
0.784	760.8905	0.1158	0.824	.7924	0.9557	0.755	.8747	0.7430
0.779	.8992	0.1190	0.874	.7993	0.9583	0.760	.8789	0.7446
0.784	.9030	0.1205	0.884	.8073	0.9613	0.751	.8858	0.7471
0.772	.9085	0.1225	0.915	.8129	0.9634	0.756	.8904	0.7489
0.772	.9127	0.1241	0.944	.8184	0.9655	0.752	.8966	0.7512
0.772	.9186	0.1263	0.976	.8243	0.9677	0.752	.9008	0.7528
0.771	.9228	0.1279	0.999	.8285	0.9693	0.758	.9067	0.7556
0.757	.9280	0.1298	1.039	.8341	0.9714	0.753	.9108	0.7565
0.769	.9321	0.1314	1.068	.8382	0.9729	0.749	.9171	0.7589
0.775	.9377	0.1335	1.121	.8462	0.9759	0.749	.9213	0.7605
0.770	.9422	0.1352	1.149	.8500	0.9773	0.756	.9282	0.7631
0.760	.9478	0.1373	1.191	.8570	0.9800	0.752	.9324	0.7646
0.769	.9519	0.1388	1.212	.8612	0.9816	0.761	.9390	0.7671
0.778	.9578	0.1410	1.259	.8671	0.9838	0.749	.9431	0.7686
0.766	.9617	0.1425	1.294	.8712	0.9853	0.749	.9497	0.7711
0.773	.9676	0.1447	1.319	.8768	0.9874	0.753	.9539	0.7727
0.771	.9710	0.1460	1.339	.8823	0.9895	0.757	.9619	0.7757
0.760	.9783	0.1487	1.380	.8882	0.9917	0.749	.9661	0.7772
0.773	.9835	0.1507	1.387	.8929	0.9934	0.773	784.8400	0.1053
+0.760	.9901	0.1531	+1.421	.9014	0.9966	+0.778	.8442	0.1069

Table Ia.—*Continued.*

V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase
$+0\overset{\rm m}{.}778$	784.8518	0.1098	$+0\overset{\rm m}{.}758$	803.7829	0.2156	$+0\overset{\rm m}{.}755$	822.8106	0.3577
0.774	.8556	0.1112	0.751	.7864	0.2169	0.758	.8162	0.3598
0.789	794.7511	0.8255	0.757	.7923	0.2191	0.755	.8221	0.3620
0.760	.7594	0.8286	0.765	.7957	0.2204	0.754	.8276	0.3641
0.764	.7636	0.8302	0.756	.8037	0.2234	0.752	.8332	0.3662
0.774	.7688	0.8321	0.763	.8075	0.2248	0.756	.8387	0.3683
0.761	.7730	0.8337	0.762	.8131	0.2269	0.751	.8443	0.3704
0.766	.7789	0.8359	0.757	.8180	0.2288	0.753	.8499	0.3725
0.780	.7830	0.8375	0.758	.8239	0.2310	0.753	.8565	0.3749
0.791	796.6987	0.5565	0.754	.8280	0.2325	0.748	.8624	0.3772
0.800	.7014	0.5576	0.764	.8343	0.2349	0.753	.8686	0.3795
0.796	.7164	0.5632	0.757	.8384	0.2364	0.759	.8742	0.3816
0.795	.7188	0.5641	0.761	.8464	0.2394	0.759	.8811	0.3842
0.795	.7313	0.5688	0.759	.8516	0.2414	0.771	834.5936	0.7805
0.795	.7334	0.5696	0.751	806.6300	0.2843	0.761	.5998	0.7828
0.786	.7459	0.5743	0.757	.6398	0.2879	0.760	.6064	0.7853
0.786	.7483	0.5752	0.759	.6439	0.2895	0.762	.6120	0.7874
0.786	.7615	0.5801	0.747	.6498	0.2917	0.760	.6175	0.7895
0.787	.7636	0.5809	0.762	.6536	0.2931	0.766	.6231	0.7916
0.778	.7778	0.5862	0.759	.6609	0.2959	0.757	.6300	0.7942
0.786	.7802	0.5871	0.744	.6679	0.2985	0.759	.6359	0.7964
0.776	803.6329	0.1593	0.752	.6745	0.3010	0.753	.6415	0.7985
0.764	.6423	0.1628	0.762	.6800	0.3030	0.757	.6481	0.8010
0.772	.6461	0.1643	0.758	.6887	0.3063	0.757	.6585	0.8049
0.765	.6520	0.1665	0.762	.6929	0.3079	0.757	.6700	0.8092
0.769	.6558	0.1680	0.760	.7002	0.3106	0.745	.6762	0.8115
0.773	.6621	0.1703	0.757	.7043	0.3121	0.756	.6825	0.8139
0.766	.6662	0.1718	0.751	822.5901	0.2749	0.757	.6884	0.8161
0.771	.6735	0.1745	0.757	.6006	0.2789	0.760	.6943	0.8183
0.766	.6770	0.1759	0.757	.6082	0.2817	0.754	.7005	0.8206
0.762	.6825	0.1779	0.764	.6144	0.2841	0.758	.7061	0.8227
0.766	.6888	0.1803	0.769	.6224	0.2871	0.760	.7130	0.8253
0.766	.6950	0.1826	0.768	.7106	0.3202	0.754	.7189	0.8275
0.761	.6985	0.1839	0.751	.7179	0.3229	0.766	.7255	0.8300
0.765	.7048	0.1863	0.760	.7235	0.3250	0.774	.7488	0.8388
0.758	.7086	0.1877	0.757	.7301	0.3275	0.768	.7543	0.8408
0.766	.7145	0.1899	0.746	.7356	0.3296	0.773	.7623	0.8438
0.767	.7183	0.1914	0.757	.7412	0.3317	0.769	.7693	0.8465
0.765	.7252	0.1939	0.748	.7467	0.3337	0.798	836.6941	0.5690
0.757	.7287	0.1953	0.756	.7523	0.3358	0.787	.7004	0.5713
0.765	.7343	0.1974	0.749	.7582	0.3380	0.789	.7059	0.5734
0.762	.7381	0.1988	0.748	.7637	0.3401	0.792	.7125	0.5759
0.764	.7454	0.2015	0.759	.7693	0.3422	0.764	855.5829	0.6589
0.764	.7489	0.2028	0.759	.7749	0.3443	0.761	.5888	0.6611
0.761	.7544	0.2049	0.748	.7821	0.3470	0.761	.5944	0.6633
0.763	.7575	0.2061	0.760	.7877	0.3491	0.765	.5999	0.6653
0.752	.7645	0.2087	0.756	.7940	0.3515	0.760	.6055	0.6674
0.756	.7680	0.2100	0.758	.7995	0.3535	0.766	.6110	0.6695
0.762	.7735	0.2121	+0.764	.8051	0.3556	+0.762	.6166	0.6716
+0.769	.7773	0.2135						

also some filamentary emission structure. It is concluded that this area of the sky shows a small amount of reddening and absorption. For the comparison stars these are taken to be $0\overset{\rm m}{.}08$ and $0\overset{\rm m}{.}25$, respectively.

From average values of Δm read from the light curves it is possible to place AS Eri on the B,V system. This variation is given in Table II for the 1956–1957 observations.

3. EPOCH AND PERIOD

The newly observed heliocentric times of minimum light are collected in Table III.

Kaho (1952) has given the best light elements to date:

$$\text{Min}=2428538.057+2.664110\,E.$$

Within the scatter of the observations all residuals from these elements lie on a straight line. From a best linear fit new elements have been determined:

$$\text{Min}_1=2428538.066+2.664152E.$$

Residuals from these elements are listed in Table III. These elements have also been used to compute the phases of the Tucson observations.

4. VARIATIONS OUTSIDE ECLIPSES

Night-by-night plots of the observations between the eclipses demonstrated that the light curves at both wavelengths changed slightly from the first season to the second. In view of the agreement in the magnitude differences between the check and comparison stars

TABLE Ib. Blue observations of AS Eridani.

V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase
$+0^{m}765$	426.7758	0.7045	$+0^{m}787$	456.7277	0.9471	$+0^{m}767$	508.6177	0.4242
0.767	.7806	0.7063	0.790	.7339	0.9494	0.780	.6233	0.4263
0.756	.7883	0.7092	0.809	.7402	0.9517	0.783	.6295	0.4286
0.747	.8022	0.7144	0.832	.7492	0.9551	0.778	.6358	0.4310
0.760	.8063	0.7160	0.926	.7700	0.9629	0.775	.6413	0.4330
0.763	.8112	0.7178	0.935	.7728	0.9640	0.781	.6486	0.4358
0.749	.8160	0.7196	0.966	.7791	0.9663	0.778	.6550	0.4382
0.751	.8258	0.7233	1.008	.7853	0.9687	0.786	.6615	0.4406
0.751	.8279	0.7241	1.108	.7999	0.9742	0.770	.6702	0.4439
0.749	.8369	0.7274	1.151	.8062	0.9765	0.782	.6760	0.4461
0.761	.8445	0.7303	1.203	.8124	0.9788	0.805	.6820	0.4483
0.782	427.7126	0.0551	1.211	.8131	0.9791	0.762	.6879	0.4505
0.789	.7237	0.0603	1.269	.8214	0.9822	0.780	.6938	0.4528
0.768	.7362	0.0650	1.321	.8277	0.9846	0.795	.6997	0.4550
0.783	.7945	0.0869	1.387	.8339	0.9869	0.780	.7066	0.4576
0.762	.8647	0.1132	1.390	.8402	0.9893	0.789	.7167	0.4613
0.771	.8785	0.1184	1.437	.8464	0.9916	0.794	.7229	0.4637
0.773	431.8670	0.6155	1.466	.8527	0.9940	0.772	513.6148	0.2999
0.774	432.6840	0.9222	1.476	.8589	0.9963	0.757	.6238	0.3032
0.792	.6861	0.9230	1.497	.8652	0.9987	0.760	.6297	0.3055
0.774	.6966	0.9269	1.502	.8714	0.0010	0.751	.6356	0.3078
0.795	.7035	0.9295	1.473	.8777	0.0034	0.757	.6415	0.3099
0.793	.7097	0.9318	1.461	.8853	0.0062	0.774	.6490	0.3127
0.782	.7160	0.9342	1.437	.8902	0.0081	0.758	.6575	0.3159
0.791	.7257	0.9378	0.771	458.6755	0.6782	0.765	514.5939	0.6674
0.780	.7347	0.9412	0.756	.6873	0.6826	0.753	.6033	0.6709
0.779	.7410	0.9436	0.761	.6935	0.6849	0.753	.6095	0.6732
0.786	.7472	0.9459	0.751	.6998	0.6873	0.753	.6151	0.6753
0.802	.7535	0.9483	0.756	.7067	0.6899	0.750	.6206	0.6774
0.799	.7597	0.9506	0.747	.7130	0.6923	0.766	.6265	0.6796
0.840	.7729	0.9555	0.756	.7199	0.6949	0.748	.6331	0.6821
0.889	.7847	0.9600	0.754	.7283	0.6980	0.760	517.6020	0.7965
0.903	.7910	0.9623	0.753	.7345	0.7003	0.761	.6109	0.7998
1.171	.8320	0.9777	0.763	.7408	0.7027	0.768	.6165	0.8019
1.221	.8396	0.9806	0.747	.7470	0.7050	0.763	518.5977	0.1702
1.283	.8486	0.9840	0.758	.7540	0.7077	0.771	.6064	0.1735
1.358	.8577	0.9874	0.759	.7651	0.7118	0.761	.6123	0.1757
1.406	.8639	0.9897	0.802	460.7941	0.4734	0.952	733.8493	0.9657
1.414	.8702	0.9921	0.801	.8045	0.4773	0.774	755.9059	0.2448
1.478	.8771	0.9947	0.812	.8108	0.4797	0.754	.9104	0.2465
1.483	.8875	0.9986	0.970	491.5929	0.0339	0.765	.9173	0.2490
1.508	.8938	0.0009	0.916	.6009	0.0369	0.772	.9215	0.2506
1.477	.9000	0.0032	0.896	.6075	0.0393	0.762	.9270	0.2527
1.459	.9070	0.0059	0.850	.6137	0.0417	0.760	.9312	0.2543
1.399	.9188	0.0103	0.839	.6200	0.0440	0.777	.9371	0.2565
1.356	.9264	0.0132	0.812	.6259	0.0462	0.770	.9413	0.2581
0.794	452.7091	0.4387	0.791	.6321	0.0486	0.752	.9472	0.2603
0.772	.7195	0.4426	0.797	.6408	0.0518	0.758	.9514	0.2618
0.790	.7300	0.4465	0.774	.6467	0.0541	0.767	.9580	0.2643
0.782	.7369	0.4491	0.797	.6526	0.0563	0.770	.9621	0.2659
0.778	.7435	0.4516	0.791	.6585	0.0585	0.767	.9677	0.2680
0.767	.7522	0.4549	0.773	.6640	0.0605	0.776	.9718	0.2695
0.795	.7605	0.4580	0.792	.6700	0.0628	0.766	.9774	0.2716
0.777	.7695	0.4613	0.789	.6759	0.0650	0.760	.9816	0.2732
0.795	.7758	0.4637	0.782	.6852	0.0685	0.768	.9892	0.2760
0.793	.7841	0.4668	0.786	.6918	0.0710	0.760	.9934	0.2776
0.799	.7904	0.4692	0.774	.6995	0.0739	0.780	756.8066	0.5828
0.800	.7994	0.4726	0.797	.7051	0.0760	0.776	.8163	0.5865
0.792	.8091	0.4762	0.777	.7137	0.0792	0.780	.8219	0.5886
0.799	.8161	0.4788	0.785	.7203	0.0817	0.775	.8274	0.5907
0.811	.8237	0.4817	0.766	.7305	0.0855	0.781	.8320	0.5924
0.804	.8320	0.4848	0.791	.7394	0.0888	0.776	.8379	0.5946
0.754	453.7695	0.8367	0.833	507.6046	0.0439	0.772	.8420	0.5961
0.765	.7778	0.8398	0.827	.6154	0.0480	0.778	.8479	0.5984
0.764	.7855	0.8427	0.775	.6216	0.0503	0.778	.8521	0.5999
0.790	456.6711	0.9258	0.791	.6452	0.0592	0.775	.8587	0.6024
0.782	.6798	0.9291	0.794	.6539	0.0624	0.770	.8629	0.6040
0.788	.6853	0.9311	0.782	.6602	0.0648	0.769	.8684	0.6060
0.785	.6916	0.9335	0.771	.6668	0.0673	0.776	.8726	0.6076
0.778	.6978	0.9358	0.780	.6751	0.0704	0.768	.8795	0.6102
0.785	.7041	0.9382	0.791	.6824	0.0731	0.777	.8837	0.6118
0.777	.7131	0.9416	0.782	508.5962	0.4161	0.778	.8893	0.6139
+0.792	.7214	0.9447	+0.760	.6070	0.4202	+0.778	.8934	0.6154

TABLE Ib.—*Continued.*

V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase
+0$^{\mathrm{m}}$772	756.8990	0.6175	+0$^{\mathrm{m}}$815	761.9506	0.5137	+0$^{\mathrm{m}}$812	769.8970	0.4964
0.779	.9031	0.6191	0.824	.9561	0.5157	0.818	.9012	0.4980
0.776	.9087	0.6212	0.816	.9607	0.5175	0.811	.9069	0.5001
0.771	.9129	0.6227	0.802	.9645	0.5189	0.814	.9109	0.5016
0.769	.9191	0.6251	0.814	.9711	0.5214	0.821	.9182	0.5044
0.771	.9233	0.6267	0.802	.9752	0.5229	0.818	.9224	0.5059
0.775	.9292	0.6289	0.803	.9811	0.5251	0.806	.9290	0.5084
0.775	.9337	0.6306	0.808	.9877	0.5276	0.817	.9332	0.5100
0.772	.9396	0.6328	0.789	.9961	0.5308	0.812	.9391	0.5122
0.774	.9438	0.6343	0.773	762.7635	0.8188	0.805	.9429	0.5136
0.777	.9500	0.6367	0.773	.7833	0.8262	0.817	.9516	0.5169
0.774	.9542	0.6383	0.771	.7864	0.8274	0.815	.9564	0.5187
0.774	.9597	0.6403	0.777	.8291	0.8434	0.807	.9630	0.5212
0.762	.9639	0.6419	0.775	.8329	0.8448	0.810	.9679	0.5230
0.775	.9695	0.6440	0.776	.8565	0.8537	0.799	.9745	0.5255
0.767	.9736	0.6455	0.777	.8607	0.8553	0.799	.9807	0.5278
0.779	.9820	0.6487	0.777	.8666	0.8575	0.797	.9873	0.5303
0.769	.9861	0.6502	0.779	.8704	0.8589	0.786	.9915	0.5319
0.775	.9917	0.6523	0.774	.8739	0.8602	0.770	783.7091	0.6808
0.759	.9958	0.6539	0.780	.9138	0.8752	0.763	.7324	0.6896
0.774	757.0020	0.6562	0.778	.9180	0.8768	0.769	.7400	0.6924
0.766	.0063	0.6578	0.773	.9242	0.8791	0.745	.7445	0.6941
0.769	759.9043	0.7456	0.775	.9284	0.8807	0.770	.7504	0.6963
0.763	.9123	0.7486	0.774	.9336	0.8826	0.767	.7539	0.6976
0.760	.9165	0.7502	0.773	.9392	0.8847	0.761	.7598	0.6998
0.759	.9227	0.7525	0.773	.9444	0.8867	0.768	.7636	0.7013
0.763	.9269	0.7541	0.769	.9485	0.8882	0.770	.7706	0.7039
0.759	.9331	0.7564	0.778	.9541	0.8903	0.768	.7740	0.7052
0.763	.9373	0.7580	0.771	.9583	0.8919	0.771	.7803	0.7075
0.765	.9432	0.7602	0.774	.9624	0.8935	0.767	.7838	0.7089
0.754	.9474	0.7618	0.777	.9683	0.8957	0.767	.7900	0.7112
0.766	.9536	0.7641	0.779	.9725	0.8972	0.773	.7938	0.7126
0.771	.9581	0.7658	0.774	.9767	0.8988	0.769	.8001	0.7150
0.762	.9668	0.7691	0.774	.9826	0.9010	0.767	.8036	0.7163
0.765	.9692	0.7700	0.788	.9867	0.9026	0.779	.8098	0.7186
0.761	.9752	0.7722	0.784	.9912	0.9043	0.765	.8136	0.7200
0.757	.9818	0.7747	0.785	.9961	0.9061	0.754	.8195	0.7223
0.768	.9884	0.7772	0.779	763.0003	0.9077	0.752	.8233	0.7237
0.758	.9946	0.7795	0.754	765.7761	0.9496	0.763	.8650	0.7393
0.783	760.8898	0.1155	0.790	.7858	0.9532	0.763	.8692	0.7409
0.772	.8985	0.1188	0.812	.7917	0.9555	0.762	.8754	0.7432
0.781	.9023	0.1202	0.835	.7976	0.9577	0.767	.8796	0.7448
0.775	.9078	0.1223	0.894	.8066	0.9611	0.762	.8865	0.7474
0.768	.9120	0.1238	0.914	.8122	0.9632	0.763	.8911	0.7491
0.772	.9179	0.1260	0.951	.8177	0.9652	0.760	.8973	0.7515
0.776	.9221	0.1276	0.987	.8236	0.9674	0.743	.9015	0.7530
0.764	.9273	0.1296	1.014	.8278	0.9690	0.758	.9074	0.7552
0.768	.9314	0.1311	1.057	.8334	0.9711	0.760	.9115	0.7568
0.768	.9370	0.1332	1.089	.8375	0.9727	0.760	.9178	0.7592
0.769	.9415	0.1349	1.148	.8455	0.9757	0.760	.9220	0.7607
0.760	.9471	0.1370	1.175	.8493	0.9771	0.762	.9289	0.7633
0.771	.9512	0.1385	1.220	.8563	0.9797	0.765	.9331	0.7649
0.784	.9571	0.1408	1.236	.8605	0.9813	0.765	.9397	0.7674
0.772	.9610	0.1422	1.296	.8664	0.9835	0.757	.9438	0.7689
0.773	.9669	0.1444	1.329	.8705	0.9850	0.753	.9504	0.7714
0.770	.9703	0.1457	1.361	.8761	0.9871	0.745	.9546	0.7730
0.766	.9766	0.1481	1.382	.8802	0.9887	0.742	.9629	0.7761
0.758	.9828	0.1504	1.430	.8875	0.9914	0.738	.9667	0.7775
0.763	.9894	0.1529	1.451	.8924	0.9933	0.756	784.8407	0.1056
0.757	.9957	0.1552	1.481	.9007	0.9964	0.755	.8449	0.1072
0.815	761.8818	0.4878	1.486	.9049	0.9980	0.789	.8511	0.1095
0.819	.8919	0.4916	1.442	.9309	0.0077	0.780	.8549	0.1109
0.813	.8961	0.4932	1.414	.9351	0.0093	0.771	794.7518	0.8257
0.820	.9016	0.4953	1.381	.9400	0.0111	0.766	.7601	0.8289
0.821	.9058	0.4969	1.348	.9466	0.0136	0.770	.7643	0.8304
0.816	.9113	0.4989	1.321	.9511	0.0153	0.767	.7695	0.8324
0.818	.9155	0.5005	1.265	.9570	0.0175	0.760	.7737	0.8340
0.822	.9197	0.5021	1.227	.9615	0.0192	0.766	.7796	0.8362
0.824	.9259	0.5044	1.170	.9677	0.0215	0.784	.7837	0.8377
0.813	.9329	0.5070	1.151	.9736	0.0237	0.793	796.6938	0.5547
0.821	.9353	0.5079	0.812	769.8759	0.4885	0.791	.7077	0.5599
0.811	.9419	0.5104	0.820	.8845	0.4917	0.796	.7108	0.5611
+0.803	.9461	0.5120	+0.811	.8901	0.4938	+0.790	.7240	0.5660

TABLE Ib.—*Continued.*

V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase	V-C	JD ⊙ 2435000+	Phase
$+0^{m}.794$	796.7261	0.5668	$+0^{m}.767$	803.8287	0.2328	$+0^{m}.783$	822.8381	0.3680
0.793	.7386	0.5715	0.769	.8350	0.2352	0.765	.8436	0.3701
0.794	.7407	0.5723	0.764	.8391	0.2367	0.765	.8492	0.3722
0.787	.7335	0.5771	0.765	.8471	0.2397	0.764	.8547	0.3743
0.785	.7559	0.5780	0.763	.8523	0.2417	0.760	.8617	0.3769
0.791	.7702	0.5834	0.742	806.6307	0.2845	0.757	.8679	0.3792
0.781	.7723	0.5842	0.756	.6404	0.2882	0.774	.8735	0.3813
0.749	803.6336	0.1596	0.761	.6446	0.2897	0.762	.8794	0.3835
0.755	.6430	0.1631	0.745	.6505	0.2920	0.782	834.5929	0.7802
0.760	.6468	0.1645	0.750	.6543	0.2934	0.765	.5988	0.7825
0.756	.6527	0.1667	0.772	.6630	0.2966	0.770	.6057	0.7851
0.757	.6565	0.1682	0.752	.6686	0.2987	0.769	.6113	0.7872
0.773	.6627	0.1705	0.749	.6762	0.3016	0.770	.6168	0.7892
0.761	.6669	0.1721	0.759	.6807	0.3033	0.781	.6224	0.7913
0.770	.6729	0.1743	0.766	.6880	0.3060	0.770	.6293	0.7939
0.775	.6777	0.1761	0.766	.6936	0.3081	0.768	.6352	0.7961
0.775	.6832	0.1782	0.753	.7009	0.3109	0.764	.6408	0.7982
0.773	.6895	0.1805	0.759	.7050	0.3124	0.777	.6474	0.8007
0.776	.6957	0.1829	0.768	822.5891	0.2746	0.778	.6578	0.8046
0.765	.6992	0.1842	0.769	.6012	0.2791	0.780	.6693	0.8089
0.771	.7055	0.1866	0.767	.6089	0.2820	0.769	.6755	0.8113
0.774	.7093	0.1880	0.762	.6137	0.2838	0.776	.6818	0.8136
0.781	.7152	0.1902	0.779	.6217	0.2868	0.783	.6877	0.8158
0.761	.7190	0.1916	0.742	.7113	0.3204	0.777	.6936	0.8180
0.777	.7259	0.1942	0.757	.7172	0.3227	0.781	.6998	0.8204
0.783	.7294	0.1955	0.768	.7228	0.3248	0.777	.7054	0.8225
0.769	.7350	0.1976	0.769	.7283	0.3268	0.767	.7123	0.8251
0.765	.7388	0.1991	0.756	.7349	0.3293	0.778	.7182	0.8273
0.773	.7461	0.2018	0.774	.7405	0.3314	0.774	.7238	0.8294
0.762	.7495	0.2031	0.759	.7460	0.3335	0.778	.7481	0.8385
0.764	.7551	0.2052	0.769	.7516	0.3356	0.770	.7536	0.8406
0.764	.7589	0.2066	0.775	.7575	0.3378	0.777	.7616	0.8436
0.768	.7652	0.2090	0.763	.7631	0.3399	0.772	.7686	0.8462
0.749	.7687	0.2103	0.780	.7686	0.3419	0.788	836.6934	0.5687
0.759	.7742	0.2123	0.769	.7742	0.3440	0.791	.6997	0.5711
0.762	.7780	0.2138	0.770	.7815	0.3468	0.792	.7052	0.5731
0.764	.7836	0.2159	0.774	.7870	0.3489	0.792	.7118	0.5756
0.760	.7871	0.2172	0.772	.7933	0.3512	0.771	855.5822	0.6587
0.762	.7930	0.2194	0.766	.7988	0.3533	0.771	.5881	0.6609
0.752	.7964	0.2207	0.763	.8044	0.3554	0.771	.5937	0.6630
0.756	.8044	0.2237	0.770	.8099	0.3574	0.766	.5992	0.6651
0.756	.8082	0.2251	0.773	.8155	0.3596	0.762	.6048	0.6672
0.766	.8138	0.2272	0.774	.8214	0.3618	0.756	.6104	0.6693
0.763	.8187	0.2290	0.765	.8269	0.3638	+0.762	.6159	0.6713
+0.756	.8246	0.2313	+0.763	.8325	0.3659			

this discordance is understood to exhibit a change in the light of at least one of the components of AS Eri.

The seasonal differences unfortunately weaken the determinations of the outside-eclipse variations, but the observations are sufficiently numerous to show a small $\cos 2\theta$ periodicity and an absence of significant variations in $\cos\theta$, $\sin\theta$, and $\sin 2\theta$. Higher harmonics also are apparently absent. The light variations are given as

$$\mathfrak{L}(y,\ 1955\text{–}56)=0.493-0.011\cos 2\theta(\pm 0^{m}.005),$$

$$\mathfrak{L}(y,\ 1956\text{–}57)=0.489-0.011\cos 2\theta(\pm 0.005),$$

$$\mathfrak{L}(b,\ 1955\text{–}56)=0.492-0.008\cos 2\theta(\pm 0.006),$$

$$\mathfrak{L}(b,\ 1956\text{–}57)=0.489-0.008\cos 2\theta(\pm 0.006).$$

The probable residuals in magnitudes from these curves for a single observation are also given.

Deviations from the computed variations in both yellow and blue light occurred during the 1956–1957 season. In particular, the yellow observations of JD

TABLE II. Magnitudes and colors of comparison stars, AS Eri.

Star	V	*B-V*	n
BD −3°576	$+7^{m}.58\pm 0^{m}.02$	$+0^{m}.14\pm 0^{m}.01$	9
BD −3°565	8.93±0.02	0.40±0.01	8
AS Eri, $0^{p}.00$	9.00	0.21	—
AS Eri, 0.25	8.33	0.15	—
AS Eri, 0.50	8.42	0.11	—
AS Eri, 0.75	+8.33	+0.15	—

TABLE III. Times of Minimum light, AS Eri.

JD ⊙ 2400000+	E	$(O\text{-}C_1)$
35432.894	+2588	$+0^{d}.003$
35456.868	2597	−0.001
35765.9102	+2713	−0.0002

2435822 running from 0ᴾ33 to 0ᴾ39 lie systematically above the curve. The interval from 0ᴾ61 to 0ᴾ67, also covered on only one night, does, however, fit the computed variation. Near the end of 1957 Dr. W. S. Fitch kindly obtained a few more observations with the original equipment. These run from 0ᴾ36 to 0ᴾ41 and depart from both the computed variation and the previous observations in this part of the light curve.

5. RECTIFICATION AND SOLUTION

The night-by-night plots of the observations also uncovered small seasonal changes within the eclipses. Within secondary eclipse the observations appear fainter in the second year; primary seems brighter by a small amount. Since the depth of secondary in 1955–1956 is unknown, a solution of the light curve can follow only from the observations of the second season. In order to inspect the changes in light, however, all observations were rectified according to

$$\mathcal{J}''(y, 1955\text{–}56)=\frac{\mathcal{L}(y, 1955\text{–}56)}{0.493-0.011\cos 2\theta}, \quad Nz=0.087,$$

$$\mathcal{J}''(y, 1956\text{–}57)=\frac{\mathcal{L}(y, 1956\text{–}57)}{0.489-0.011\cos 2\theta}, \quad Nz=0.088,$$

$$\mathcal{J}''(b, 1955\text{–}56)=\frac{\mathcal{L}(b, 1955\text{–}56)}{0.492-0.008\cos 2\theta}, \quad Nz=0.064,$$

$$\mathcal{J}''(b, 1956\text{–}57)=\frac{\mathcal{L}(b, 1956\text{–}57)}{0.489-0.008\cos 2\theta}, \quad Nz=0.064.$$

In a case such as AS Eri a solution from the Princeton nomographs is ambiguous, at least at the beginning. This arises because it is very difficult to determine the shape of secondary and thus distinguish which eclipse is the transit and which is the occultation. Both possibilities must be investigated, and the best hope for a conclusive discrimination lies along the upper parts of the branches of primary eclipse. If also the eclipses are of different depths at the two wavelengths, the intersection of the two depth lines should help resolve the question.

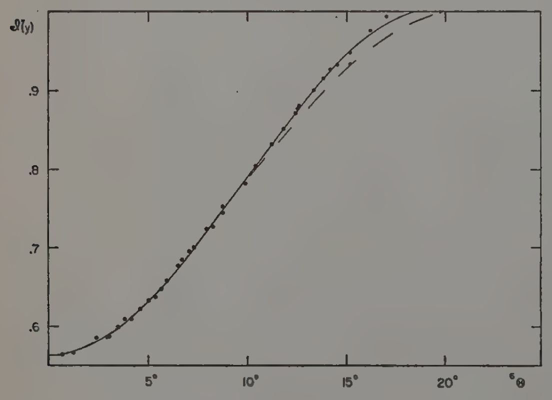


FIG. 3. Rectified yellow observations of 1956–1957 season within primary eclipse. The solid curve is computed from the solution of Table IV. The broken curve is the parasitic solution in which primary eclipse would be annular.

TABLE IV. Solution of light curves, AS Eri.

$x=0.6$
$k=0.787$
$a_g=0.204$
$b_g=0.201$
$a_s=0.160$
$b_s=0.158$
$z_y=0.034$
$z_b=0.025$
$j=79^\circ.6$

$\alpha_0^{oc}=\alpha_0^{pr}=0.503$
$\alpha_0^{tr}=\alpha_0^{sc}=0.454$
$D=0^d.273$
$L_{g,y}=0.132$
$L_{s,y}=0.868$
$L_{g,b}=0.066$
$L_{g,b}=0.934$
$\langle (J_s/J_g)_y\rangle_{av}=10.6$
$\langle (J_s/J_g)_b\rangle_{av}=22.7$

For AS Eri the two possibilities may be described as follows: (1) primary is an annular eclipse of the A star by a cooler dwarf—essentially the solution given by Gaposchkin; or (2) the eclipses are partial, and primary is an occultation of the A star by a cooler, larger one. For limb darkening 0.6 the two solutions are plotted in Fig. 3 against the yellow observations of the second season. It is apparent that the solution for partial eclipses is preferable, and this conclusion is substantiated by a similar test of the blue observations. In addition, the partial eclipse curve fits the observations of 1955–1956 more satisfactorily than the complete eclipse curve does. Further trials proved that different values of the mean darkening coefficient could not make an arguable case for complete eclipses.

For the mean darkening coefficient, values of 0.2, 0.4, 0.6, and 0.8 were investigated. While there is relatively little change from one darkening to another, a value of $x=0.6$ seemed to provide the best common solution for both yellow and blue. This solution is given in Table IV.

From this solution light curves were computed and are plotted in Figs. 1 and 2. From these plots magnitude differences were read off at convenient intervals in order to compile the light curves of Table V. Within the minima the probable residual for a single observation from the computed curve is ±0ᵐ004 in yellow and ±0ᵐ005 in blue.

The solution was examined both in yellow and blue light to see if different darkenings were justified for the two eclipses. No conclusive evidence in favor of this elaboration could be found.

The observations of the first season must be reconciled with this solution. If within primary these observations are rectified using the coefficients from the 1956–1957 season, they fit the light curve computed from the best solution reasonably well. If however, they are rectified according to the outside-eclipse variation from the 1955–1956 season, they cannot be made to fit unless the brightness of at least one of the components has changed or a small amount of additional light was present during the first season perhaps in the form of

TABLE V. Computed light curves, AS Eri.

Phase	Δm_y	Δm_b	Phase	Δm_y	Δm_b
±0.000	$+1^{m}.425$	$+1^{m}.484$	±0.300	$+0^{m}.757$	$+0^{m}.762$
0.005	1.406	1.461	0.350	0.769	0.771
0.010	1.354	1.404	0.400	0.784	0.782
0.015	1.280	1.320	0.450	0.796	0.791
0.020	1.192	1.222	0.455	0.800	0.793
0.025	1.105	1.121	0.460	0.804	0.796
0.030	1.014	1.026	0.465	0.811	0.799
0.035	0.936	0.943	0.470	0.817	0.802
0.040	0.873	0.873	0.475	0.824	0.805
0.045	0.826	0.822	0.480	0.831	0.809
0.050	0.798	0.794	0.485	0.836	0.812
0.100	0.784	0.782	0.490	0.841	0.815
0.150	0.769	0.771	0.495	0.845	0.816
0.200	0.757	0.762	±0.500	+0.846	+0.817
±0.250	+0.753	+0.759			

a gas stream. It is impossible to choose between these possibilities.

6. DISCUSSION

Gaposchkin gives $f(\mathcal{M})=0.0029\odot$. In many systems showing similarly small mass functions, the less massive component has been found to approach the Roche limit characteristic of the binary. For AS Eri this occurs if $(\mathcal{M}_h/\mathcal{M}_c)=10$. If this value is assumed, $\mathcal{M}_h=3.7\odot$, $\mathcal{M}_c=0.37\odot$, $\mathcal{R}_h=2.1\odot$, and $\mathcal{R}_c=1.8\odot$. The mass and radius of the hot star do not conform to the mass-radius and mass-luminosity relations as given, for instance, by Kopal (1959). Fair agreement with these relations is possible with the following: $\mathcal{M}_h=1.4\odot$, $\mathcal{M}_c=0.20\odot$, $\mathcal{R}_h=1.5\odot$, and $\mathcal{R}_c=1.9\odot$. The system is now detached.

From the tie-in to the *B,V* system and the solution of the light curves the apparent magnitudes and colors of the components can be derived. The results are given in Table VI. According to Johnson (1955) some of the nearby A0V stars range in *B-V* from $-0^{m}.03$ to $+0^{m}.04$. AS Eri then appears reddened by about $0^{m}.06$. When corrected for this amount of reddening, the *B-V* index of the cool star is $+0^{m}.91$. Stars of this color index occur among classes G8 III, G8 IV, and K2 V. The mass and radius of the cool star derived above indicate that G8 IV is the most suitable classification. Effective temperatures of 10 700°K and 4875°K are assumed for the hot and cool components, respectively. From these and their radii in solar units the following bolometric magnitudes are computed: $M_{\mathrm{bol},h}=+1^{m}.00$ and $M_{\mathrm{bol},c}=+3^{m}.22$. The magnitude and color of the hot star place it close to the initial main sequence tabulated by Schwarzschild (1958). Bolometric corrections of $0^{m}.72$ and $0^{m}.32$ are assumed for the hot and cool star, respectively, and the observed visual magnitudes given in Table VI are corrected by $0^{m}.18$ for interstellar absorption. The distance of AS Eri from the sun can then be computed for each component individually. From the data for the hot star the distance is 205 parsecs while a similar treatment for the cool star yields 226 parsecs. The difference is not significant, for errors in the spectral classifications, magnitudes, and bolometric corrections are certainly present. A distance of about 200 parsecs is, however, strongly indicated.

TABLE VI. Magnitudes and colors of components of AS Eri.

	Star_h	Star_c
V	$+8^{m}.46$	$+10^{m}.47$
B-V	+0.06	+ 0.97

Kopal gives the annual proper motions as

$$\mu_\alpha=-0''.031\pm0''.010 \quad \text{and} \quad \mu_\delta=+0''.026\pm0''.012,$$

and Gaposchkin lists $V_\gamma=+40$ km/sec. Corrected for the standard solar motion and apex these values and a distance of 200 parsecs determine a space velocity of 46 km/sec. This determination is obviously quite uncertain.

7. SUMMARY AND ACKNOWLEDGMENTS

From the study of numerous photoelectric observations it has become possible to reject the old photometric solution and derive new elements which describe the system as probably detached. A new series of high-dispersion spectrograms is in order, particularly to verify the mass function.

This paper, now somewhat revised, was received by the University of Pennsylvania in partial fulfillment of the requirements for the doctor's degree in astronomy. I would like to acknowledge my gratitude to Professors E. F. Carpenter and W. S. Fitch of the Steward Observatory for permitting me the use of the Observatory's facilities. At the University of Pennsylvania Drs. F. B. Wood, J. E. Merrill, L. Binnendijk, W. Blitzstein, W. M. Protheroe, and K. T. Menon kindly offered assistance and advice. Fellowships from the University of Arizona and the University of Pennsylvania are gratefully acknowledged.

REFERENCES

Gaposchkin, S. 1946, *Harvard Bull.* **918**, 12.
——. 1953, *Harvard Ann.* **113**, 108.
Hoffmeister, C. 1934, *Astron. Nachr.* **253**, 195.
Huang, Su-Shu and Struve, O. 1956, *Astron. J.* **61**, 300.
Hubble, E. 1934, *Astrophys. J.* **79**, 55.
Johnson, H. L. 1955, *An. d'Ap.* **18**, 292.
Kaho, S. 1952, *Tokyo Astr. Bull.* **49**, 385.
Kopal, Z. 1942, *Harvard Bull.* **916**, 16.
——. 1959, *Close Binary Systems* (Belfast University Press, Belfast, Ireland), p. 486.
Schwarzschild, M. 1958, *Structure and Evolution of the Stars* (Princeton University Press, Princeton, New Jersey), p. 10.

THE ASTRONOMICAL JOURNAL VOLUME 65, NUMBER 3 APRIL, 1960

Parallax, Proper Motion, and Mass Ratio of Σ2398 (ADS 11632)

Heinrich Eichhorn* and Harold L. Alden
Leander McCormick Observatory, University of Virginia, Charlottesville, Virginia

(Revised draft received December 8, 1959)

From the 92 McCormick plates of the double star Σ 2398 (1900: R.A. $18^h41^m.7$, Dec. $+59°29'$) which have accumulated since 1919, parallax and proper motion were newly determined. The McCormick material alone does not permit an accurate determination of the mass ratio. Therefore a series of Yerkes 40-in. plates, taken 1902–1904, and an older series of visual observations by Lamp were reduced to the McCormick system and included in our mass-ratio solution. However, the accuracy of the reduction of one series to another is at present still severely limited by yet unknown systematic effects in the positions of the comparison stars. Thus, a method is suggested and applied of exploiting as much as possible the high intrinsic accuracy of the Yerkes series by using the coordinate differences and proper motions in the solution for the mass ratio.

For the relative parallax we obtain $+0''.287\pm0''.005$, for the relative proper motion of the center of gravity $\mu_\alpha \cos\delta = -1''.3188 \pm0''.0004$ and $\mu_\delta = +1''.8347\pm0''.0003$, and for $\mathfrak{M}_2:\mathfrak{M}_1$, the mass ratio, 0.996 ± 0.074. The residuals in both coordinates suggest systematic perturbations in the motions of both components.

1. DETERMINATION OF MASS RATIOS FROM SHORT ARCS

It is frequently desirable to determine the mass ratio of a double star in the case where the orbit segments of the star's components are not sufficiently well defined by the observations to permit a determination of orbital elements.

Throughout the paper subscripts 1 and 2 will refer to the brighter star π_1, with the mass $\mathfrak{M}_1$ and the companion π_2 with the mass $\mathfrak{M}_2$, respectively; unprimed symbols refer to the standard coordinate x, primed symbols to the standard coordinate y.

Assume μ and μ' to be the components of the proper motion of the center of gravity of the system. We assume that the nonlinear terms of this proper motion are negligible. The components ξ_2 and ξ_2' of the distance of π_2 from the center of gravity are analytical functions of the time t and can therefore be expanded as power series

$$\xi_2=\sum_{k=0}^{\infty} a_k(E)t^k,$$

$$\xi_2'=\sum_{k=0}^{\infty} a_k'(E)t^k,$$

in which the coefficients a_k and a_k' are functions of the elements, which are symbolically written as E. If λ is the mass ratio $\mathfrak{M}_2:\mathfrak{M}_1$, we have then for the "plate solutions" m and m',

$$m_1=c_1+\mu t+P\pi-\lambda \sum a_k t^k,$$
$$m_2=c_2+\mu t+P\pi+\sum a_k t^k,$$
$$m_1'=c_1'+\mu' t+P'\pi-\lambda \sum a_k' t^k,$$
$$m_2'=c_2'+\mu' t+P'\pi+\sum a_k' t^k.$$

π is the parallax of the system; P and P' are the parallax factors. Only seven of the coefficients a_k and a_k' are independent. In practice, these series have to be broken off after a finite number of terms; in the simplest case the arcs can be described by the section of a parabola as well as by anything else. In this case the retention of higher-order terms than t^2 would only increase the systematic errors. An analysis in the form

$$m_1=c_1-\lambda a_0+t(\mu-\lambda a_1)-t^2\lambda a_2+P\pi,$$
$$m_2=c_2+a_0+t(\mu+a_1)+t^2a_2+P\pi;$$

or, denoting the unknowns we have to solve for by b_{ik}

$$m_1=b_{10}+tb_{11}+t^2b_{12}+P\pi,$$
$$m_2=b_{20}+tb_{21}+t^2b_{22}+P\pi, \qquad (1)$$

and analogously for the m_1' and m_2' components, will give the parallax of the system, position, and proper motion of the center of gravity, and the mass ratio. In general (i.e., whether or not the series are broken off with t^2), the freedom of the unknowns is restricted as a consequence of the sequence of equations between the the a_k and a_k' that originate, among other causes, from the area theorem

$$\xi(d\xi'/dt)-\xi'(d\xi/dt)=f,$$

if all the coefficients of t^i $(i>0)$ are put equal to zero, and from the requirement that

$$b_{1k}/b_{2k}=b_{1k}'/b_{2k}'=\lambda \quad \text{for all } k>1.$$

Equations (1) are not the only relations which can be used as observation equations for the computations of the unknowns. In general observation equations between the unknowns are also provided by the equations which are satisfied by the components of the proper motions of the stars at a certain time

$$\mu_{\pi_1}=\mu-\lambda a_1-2t\lambda a_2+\cdots,$$
$$\mu_{\pi_2}=\mu+a_1+2ta_2+\cdots \qquad (2)$$

(similarly for the y components), and also by those originating from the components of the relative positions of the pair

---

* Now at Van Vleck Observatory, Wesleyan University, Middletown, Connecticut.

$$\Delta\xi \equiv m_1-m_2=c_1-c_2-a_0(1+\lambda) -ta_1(1+\lambda)-t^2a_2(1+\lambda)-\cdots \qquad (3)$$

(analogously for $\Delta\xi'$).

For a definitive treatment it would be desirable to include, besides all available parallax series, the better series of relative coordinate measures, and to solve for orbital elements, parallax, proper motion, and mass ratio simultaneously, imposing all conditions on the parameters that exist.

Those plate solutions m and m', which originate from different parallax series will generally be referred to different systems of reference stars. In order to combine them in one solution, the relations between the reference systems must be established and all data reduced to a common system. This is not always possible with the desired exactness, as it requires a very accurate knowledge of the relative positions of the reference stars and because parallax series as a rule are measured purely differentially so that systematic errors, which are difficult to ascertain, will generally remain. These systematic errors are far less dangerous for the reduction of proper motions because these can be determined from differential measurements within one system. Relative coordinates in most cases require no reduction from one system to another.

2. McCORMICK MATERIAL

For Σ2398, Table I establishes the McCormick reference system used for the combination into a common solution of results from different series of observations. For 21 stars in the field around this pair, it provides AC Vatican and BD numbers, photographic magnitudes, uncorrected nearly-rectangular coordinates in millimeters for the epoch 1951.69 oriented approximately to the equator of 1900, and annual proper motions (pe about $\pm 0''.002$) relative to the mean of all these stars with the exception of those with an asterisk.

Σ2398 was put on the McCormick parallax program in 1919. The components ($\pi_1: dM_4$, $m_{\mathrm{vis}}=8^{\mathrm{m}}.96$; $\pi_2: dM_4$, $m_{\mathrm{vis}}=9^{\mathrm{m}}.69$) of this system will eventually be among those with the most accurately determined masses because of its large parallax in connection with its large semi-major axis, which has up to now prevented several computers [Wieth-Knudsen (1953), Rabe (1954), Hopmann (1954), and Guentzel-Lingner (1955)] from finding a satisfactory orbit; these authors suggest periods between infinity and about 300 years.

The 92 parallax plates, taken with the 26-in. McCormick refractor with sector opening 40%, were measured in the usual way in both coordinates. Stars 5, 7, 11, and 15 of Table I were used as comparison stars, the dependences being 0.234, 0.250, 0.266, and 0.250, respectively. The orientation of all plates had to be sufficiently accurate so as not to introduce any errors, as the proper motion of the pair is large, and the same set of dependences was used throughout.

The slight change of orientation with time, brought about by the small proper motion of the orienting stars (8 to 11 in x, 5 and 15 in y), introduces an artificial change of the measured proper motion of the pair, which may be neglected in the present case as the intrinsic

TABLE I. The McCormick system.

No.	AC Vatican	BD	m_{pg}	x (mm)	y (mm)	$\mu_\alpha \cos\delta$	μ_δ
1	47755	+59°1912	$9^{\mathrm{m}}.7$	33.9412	89.9182	$-0''.002$	$-0''.004$
2	47838		10.1	36.0868	158.4278	−0.011	+0.016
3	47793		10.4	38.4822	117.0982	+0.002	+0.009
4	47738		10.4	40.6852	71.8145	−0.002	−0.007
5	47739		10.4	40.8406	74.6425	+0.007	+0.002
6*	47774	+59 1913	8.8	45.7354	104.5753	−0.022	−0.050
7	47820		10.4	62.0771	140.8531	+0.005	−0.002
8	47728	+59 1914	8.8	79.3775	61.9348	0.000	−0.004
9	47808		10.4	80.9460	134.8648	+0.002	−0.045
10	47840		9.7	81.4048	157.4990	+0.001	−0.011
11	47822		10.9	92.4008	146.1749	−0.001	−0.001
12	47794	+59 1916	10.4	93.2574	122.8076	0.000	−0.006
13	47740	+59 1917	10.4	105.3114	80.6082	−0.012	+0.007
14	47741	+59 1918	8.8	133.4301	80.5484	+0.007	+0.012
15	47744	+59 1920	10.1	150.3132	71.8127	+0.001	−0.002
16	47779	+59 1919	7.6	150.7155	102.3718	−0.001	−0.004
17*	47795	+59 1921	7.6	154.3225	124.4265	−0.058	−0.039
18	47796		11.3	156.2480	118.5455	+0.004	−0.004
19*	47759		10.4	159.3452	85.8630	−0.052	+0.010
20*	47746		10.4	170.0250	73.7712	−0.054	+0.018
21*	47729	+59 1922	7.6	174.1525	62.5645	+0.042	+0.045
π_1	47777	+59 1915	8.8	86.0810	111.3612		
π_2	47778		10.4	86.3322	110.6435		

TABLE II. McCormick observations.

Ep	Pl	Exp	Wt	P	P'	t	m_1	v_1	m_2	v_2	m_1'	v_1'	m_2'	v_2'
1	4	8	3.6	+0.64	+0.77	−44.99	−198	+2	+232	+19	−46	−38	−99	−38
2	5	10	4.6	−0.64	+0.73	−44.15	−199	−4	+198	−11	+18	+25	−49	+12
3	4	8	3.2	+0.67	+0.71	−41.37	−176	+9	+202	+6	−5	0	−78	−15
4	2	4	1.2	−0.74	+0.63	−40.41	−180	−1	+206	+12	−2	+2	−75	−10
5	2	4	1.6	+0.68	+0.74	−37.71	−206	−36	+129	−50	−14	−11	−71	−6
6	3	6	2.4	−0.78	+0.61	−36.72	−142	+21	+170	−9	−12	−10	−81	−15
7	3	6	2.0	+0.57	+0.82	−33.99	−148	+6	+136	−28	+4	+4	−77	−10
8	2	4	1.2	−0.16	+0.99	−18.91	−83	+3	+120	+19	−2	−9	−77	−4
9	1	2	1.0	+0.25	+0.97	−11.85	−70	−14	+59	−9	+24	+16	−88	−15
10	3	6	2.2	−0.05	+1.01	−11.67	−25	+30	+83	+14	−4	−11	−88	−15
11	2	4	2.0	−0.18	+0.99	−0.65	+21	+26	+54	+34	+17	+10	−63	+8
12	4	8	3.8	−0.63	+0.63	+14.32	+60	−4	−33	+16	−6	−8	−66	−3
13	2	4	2.0	+0.59	+0.80	+17.14	+34	−40	−60	+4	−8	−9	−38	+24
14	1	2	0.8	−0.99	+0.17	+18.34	+11	−72	−119	−52	−30	−31	−16	+44
15	2	4	1.6	+0.95	+0.28	+20.41	+88	−1	−108	−28	+28	+27	+17	+76
16	4	8	4.0	+0.58	+0.82	+28.10	+140	+14	−117	−1	+26	+32	−38	+14
17	2	4	2.0	−0.93	+0.38	+29.18	+110	−23	−133	−14	+12	+19	−38	+12
18	2	4	1.2	−0.95	+0.33	+32.86	+120	+60	−114	+22	−28	+18	−62	−17
19	4	7	3.0	+0.70	+0.70	+35.31	+156	−4	−142	+9	−1	+12	−12	+32
20	4	8	4.0	−0.82	+0.59	+36.34	+173	+6	−152	+1	−23	−8	−46	−2
21	2	6	2.4	+0.48	+0.88	+72.00	+334	−4	−342	−10	−46	+22	+12	−1
22	4	13	5.0	−0.92	+0.40	+72.99	+340	−5	−353	−18	−55	+14	+35	+18
23	4	12	4.8	+0.54	+0.82	+75.60	+339	−16	−375	−22	−66	+9	+34	+12
24	5	15	6.0	−0.67	+0.67	+76.43	+370	+8	−360	−7	−81	−4	+24	0
25	3	11	3.6	+0.60	+0.77	+79.22	+357	−16	−382	−12	−88	−6	+17	−12
26	4	13	4.8	−0.68	+0.74	+80.05	+355	−24	−365	+6	−102	−18	+11	−20
27	2	6	2.4	+0.90	+0.42	+82.59	+404	+14	−344	+44	−68	+21	+54	+16
28	4	12	4.8	−0.60	+0.82	+83.64	+406	+9	−383	+8	−95	−3	+17	−23
29	4	16	5.6	+0.86	+0.49	+86.28	+419	+11	−395	+12	−104	−7	+50	+4
30	4	16	5.4	−0.70	+0.72	+87.37	+428	+12	−404	+6	−118	−16	+34	−14

change of the proper motions of the components due to orbital motion exceeds this effect by a factor of about 100.

Table II gives the relevant information for the McCormick measures. We have removed from the straight plate solutions approximate solutions as follows:

$$m_1 = m_{\text{obs}} - (+1000 - 174\,t + 140\,P);$$

$$m_2 = m_{\text{obs}} - (+4200 - 174\,t + 140\,P);$$

$$m_1' = m_{\text{obs}} - (+700 + 244\,t + 140\,P');$$

$$m_2' = m_{\text{obs}} - (-6600 + 244\,t + 140\,P').$$

The unit for all measures is 0.0001 mm on the plates. One millimeter corresponds to 20″.75. The time t is in units of 100 days counted from JD 2426600.0 = 1931.71. For the individual observations the time was used to the tenth of a day to avoid the systematic effect on the parallax that would have been introduced by rounding off the time to the nearest day. In all the computations, seasonal means have been used. The v are the seasonal means from the finally adopted solutions: (b) for x, (d′) for y.

The relative coordinates of the components are given in Table III referred to the equator of 1900.0, together with the relative weights. A least-squares solution was made for the coefficients b_{ik} and π of Eqs. (1) from the material in Table II. We allowed different results for the mass ratio from the x and the y measures and dif-

TABLE III. Relative positions of components.

t	Δx	Δy	Wt
1919.39	+7″.53	−15″.26	3.6
1919.62	7.46	15.29	4.6
1920.38	7.42	15.30	3.2
1920.65	7.44	15.30	1.2
1921.39	7.33	15.27	1.6
1921.66	7.29	15.29	2.4
1922.40	7.23	15.32	2.0
1926.53	7.06	15.30	1.2
1928.47	6.91	15.38	1.0
1928.51	6.86	15.32	2.2
1931.53	6.71	15.31	2.0
1935.63	6.45	15.27	3.8
1936.40	6.44	15.21	2.0
1936.73	6.37	15.12	0.8
1937.30	6.23	15.17	1.6
1938.40	6.11	15.28	4.0
1939.70	6.13	15.25	2.0
1940.71	5.97	15.22	1.2
1941.38	6.02	15.17	3.0
1941.66	5.97	15.21	4.0
1951.42	5.24	15.03	2.4
1951.69	5.20	14.96	5.0
1952.41	5.16	14.94	4.8
1952.64	5.13	14.93	6.0
1953.40	5.11	14.93	3.6
1953.63	5.15	14.91	4.8
1954.32	5.09	14.89	2.4
1954.61	5.00	14.92	4.8
1955.33	4.95	14.83	5.6
1955.63	+4.91	−14.83	5.4

TABLE IV. The Yerkes-Yale observations in the McCormick system.

Pl	Exp	t	m_1	v_1	m_2	v_2	m_1'	v_1'	m_2'	v_2'
22	54	−98.94	−335	+83	+510	+84	−114	−23	+6	−23

ferent values for the parallax from both components in both coordinates. The results were in x:

$$\begin{aligned} c_1&=+0.0997\pm4\ \text{mm}\\ \mu_1&=-0.0169535\pm92\ \text{mm}/100\ \text{d}\\ &=-1''.2849\pm7/\text{yr}\\ b_{12}&=+0.00000037\pm17\\ \pi&=+0''.284\pm8\\ \text{pe }1&=\pm0.00225\ \text{mm}=\pm0''.046\\ c_2&=+0.4215\pm4\ \text{mm}\\ \mu_2&=-0.0178526\pm92\ \text{mm}/100\ \text{d}\\ &=-1''.3530\pm7/\text{yr}\\ b_{22}&=-0.00000039\pm17\\ \pi&=+0''.290\pm8\\ \text{pe }1&=\pm0.00225\ \text{mm}=\pm0''.046 \end{aligned}\tag{a}$$

and in y:

$$\begin{aligned} c_1'&=+0.0717\pm9\ \text{mm}\\ \mu_1'&=+0.0243939\pm74\ \text{mm}/100\ \text{d}\\ &=+1''.8488\pm6/\text{yr}\\ b_{12}'&=-0.00000148\pm13\\ \pi&=+0.280\pm25\\ \text{pe }1&=\pm0.00194\ \text{mm}=\pm0''.040\\ c_2'&=-0.6629\pm9\ \text{mm}\\ \mu_2'&=+0.0244577\pm74\ \text{mm}/100\ \text{d}\\ &=+1''.8536\pm6/\text{yr}\\ b_{22}'&=+0.00000065\pm13\\ \pi&=+0''.198\pm25\\ \text{pe }1&=\pm0.00194\ \text{mm}=\pm0''.040. \end{aligned}\tag{a$'$}$$

(Instead of b_{i0} abd b_{i1} we have written the conventional notations c_i and μ_i.) The probable errors of unit weight were computed from the seasonal residuals rather than the residuals from the individual plates; the latter would have given somewhat smaller values. Throughout the paper the errors are given in units of the last figure of the corresponding quantity.

3. YERKES-YALE AND LAMP'S OBSERVATIONS

To strengthen the mass ratio, we incorporated two more series of measures into our solution.

Brouwer and Jenkins (1946) used stars 6, 12, 13, and 14 from Table I for the reduction of Yerkes parallax plates. The solutions which they communicate can be reduced to the McCormick system by means of Table I, and give the data in Tables IV and V. The determination of the values in the x coordinate of Table IV (m_1, m_2) was of such low weight that they were discarded. The reasons for this are probably systematic differences between the McCormick and the Yerkes-Yale measures which could not be established because only the four Yerkes reference stars were available to fit the Yerkes measures into the McCormick system. The weight 2.0 was assigned to the y values of Table IV, (m_1', m_2').

We may assume that the material in Table V suffers no appreciable loss of weight by the reduction to the McCormick system; the weights for these data were computed from the errors quoted by Brouwer and Jenkins on the basis of a probable error of 0.0021 mm for unit weight and rounded to the next exact square figure for reasons of computational convenience.

The data from Tables IV and V were now included into our solution, the observation equations for the latter being patterned after Eqs. (2) and (3). At this point it was assumed that the two components should yield the same parallax in a given coordinate, but the y coordinate was still treated separately from x. The following figures resulted from this analysis, in x:

$$\begin{aligned} c_1&=+0.0998\pm4\ \text{mm}\\ \mu_1&=-0.0169491\pm57\ \text{mm}/100\ \text{d}\\ &=-1''.2846\pm4/\text{yr}\\ b_{12}&=+0.00000031\pm10\\ \pi&=+0''.287\pm5\\ c_2&=+0.4216\pm4\ \text{mm}\\ \mu_2&=-0.0178539\pm57\ \text{mm}/100\ \text{d}\\ &=-1''.3531\pm4/\text{yr}\\ b_{22}&=-0.00000039\pm10\\ \text{pe }1&=\pm0.0022\ \text{mm}=\pm0''.046 \end{aligned}\tag{b}$$

and in y:

$$\begin{aligned} c_1'&=+0.0732\pm7\ \text{mm}\\ \mu_1'&=+0.0243955\pm38\ \text{mm}/100\ \text{d}\\ &=+1''.8489\pm3/\text{yr}\\ b_{12}'&=-0.000001529\pm71\\ \pi&=+0''.237\pm16\\ c_2'&=+0.6647\pm7\ \text{mm}\\ \mu_2'&=+0.0244459\pm38\ \text{mm}/100\ \text{d}\\ &=+1''.8528\pm3/\text{yr}\\ b_{22}'&=+0.000000895\pm71\\ \text{pe }1&=\pm0.020\ \text{mm}=\pm0''.41. \end{aligned}\tag{b$'$}$$

TABLE V. Yerkes-Yale proper motions and coordinate differences in the McCormick system.

	Wt	Value		Residual	
μ_x, π_1	676	$+0.000282$	mm/100d	$+0.000108$	
μ_y, π_1	676	$+$ 316	"	$-$ 95	
μ_x, π_2	676	$-$ 181	"	$-$ 196	Epoch 1904.64
μ_y, π_2	676	$-$ 460	"	$-$ 96	$(t=-98.94)$
Δx	100	-0.0845	mm	-0.00001	
$+\Delta y$	144	-0.0108	mm	$+0.00001$	

The residuals in the y coordinate in Tables IV and V were computed from solution (b).

As the correction to the parallax from the x measures carries so much more weight than that from the y measures, the latter was disregarded. Therefore we get from the McCormick measures as final result for the parallax

$$\pi=+0''.287\pm0''.005 \text{ pe.}$$

The mass ratio from x results $\lambda=0.79\pm0.33$ and from y, $\lambda'=1.71\pm0.16$. This finishes the discussion of the available data in x and the values from (b) served as final solution for the computation of all the x residuals given in this paper.

E. Lamp devoted two papers to the determination of this star's parallax. In the first, Lamp (1885) observed visually with a wire micrometer, mounted on the Steinheil refractor (aperture 208 mm, focal length 3300 mm) of the Kiel Observatory, declination differences of π_1 compared to stars 6 and 16 from Table I. In the second paper he communicates declination differences of π_1 and π_2 compared to these two stars and BD $+59°1911$. The latter is unfortunately too close to the edge of the McCormick plates to permit an accurate determination of its position and proper motion, and could therefore not be utilized for the reduction of Lamp's observations to the McCormick system.

Table VI gives the result of the reduction of Lamp's data to the McCormick system, grouped in four normal positions for π_1 and two normal positions for π_2, in analogy with Table II. The first column gives the number of the normal place. In the second column is the number of included observations. The weights in the third are roughly proportional to this number. Higher total weights than 0.6 for π_1 and 0.3 for π_2 would not have been justified because of the uncertainties inherent in the reduction to the McCormick system and Lamp's observations themselves.

TABLE VI. The reduced Lamp data.

Normal position No.	Obs	Wt	P'	t	m_1'	v_1'
1	21	0.10	0.00	-175.72	-249	$+83$
2	31	0.15	0.00	-171.81	-216	$+100$
3	57	0.25	0.00	-168.19	-228	$+74$
4	27	0.12	0.00	-163.95	-204	$+82$

Normal position No.	Obs	Wt	P'	t	m_2'	v_2'
1	46	0.20	0.00	-167.88	$+270$	$+45$
2	27	0.10	0.00	-163.95	$+442$	$+241$

Finally these reduced Lamp data from Table VI were incorporated into the normal equations for y. Two solutions were made. The first allows for a correction to the assumed value of the parallax to be determined from the y measures. We get

$$\begin{aligned} c_1'&=+0.0725\pm8 \text{ mm}\\ \mu_1'&=+0.0243845\pm33 \text{ mm/100 d}\\ &=1''.8481\pm3/\text{yr}\\ b_{12}'&=-0.000001259\pm72\\ \pi&=+0''.241\pm18 && \text{(c}'\text{)}\\ c_2'&=-0.6653\pm8 \text{ mm}\\ \mu_2'&=+0.0244334\pm33 \text{ mm/100 d}\\ &=+1''.8515\pm3/\text{yr}\\ b_{22}'&=+0.000001146\pm69\\ \text{pe } 1&=\pm0.0023 \text{ mm}=\pm0''.048 \end{aligned}$$

from which $\lambda'=1.10\pm0.09$. If a second solution is made assuming $\pi=0''.2872$ (corresponding to $\Delta\pi=$ -0.0001593), which is our finally adopted value, we get

$$\begin{aligned} c_1'&=+0.0709\pm4 \text{ mm}\\ \mu_1'&=+0.0243817\pm38 \text{ mm/100 d}\\ &=+1''.8479\pm3/\text{yr}\\ b_{12}'&=-0.000001207\pm62\\ c_2'&=-0.6669\pm4 \text{ mm} && \text{(d}'\text{)}\\ \mu_2'&=+0.0244316\pm37 \text{ mm/100 d}\\ &=+1''.8514\pm3/\text{yr}\\ b_{22}'&=+0.000001199\pm66\\ \text{pe } 1&=\pm0.0023 \text{ mm}=\pm0''.048 \end{aligned}$$

from which follows $\lambda'=1.007\pm0.076$. We regard (d′) as giving the final values from the analysis of the y data and all the primed residuals in this paper were computed from this solution. The weighted mean for the mass ratio from x and y data gives

$$\mathfrak{M}_2:\mathfrak{M}_1=0.996\pm0.074.$$

This is the final result for the mass ratio from this paper. It agrees well enough with the value 0.83 that Hopmann (1954) obtained from the mass-luminosity relation.

If the residuals are plotted against the time there are apparently still systematic and likely periodic errors left for which we cannot account. These residuals are similar for both components. By comparing with the residuals of an unpublished investigation of parallax, proper motion, and mass ratio of this star (Garceau 1951), no equally long periods for x and y can be established although a period in x of about $4\frac{1}{2}$ yr and one in y of somewhat like 9 yr, both with an amplitude in the order of $0''.03$, seem to fit. A decision as to the reality of the systematic and perhaps periodic nature of these residuals must be postponed until more material has accumulated.

ACKNOWLEDGMENT

One of us (H.E.) is greatly indebted to the International Cooperation Administration, Washington, D.C., for awarding him a fellowship upon recommendation of the National Academy of Sciences, Washington, D.C., during the tenure of which his share of this work was done. We are also indebted to Dr. Peter van de Kamp for putting at our disposal Miss Garceau's unpublished investigation.

REFERENCES

Brouwer, D. and Jenkins, L. F. 1946, *Astron. J.* **51**, 54.
Garceau, C. L. 1951 (*Archives of Sproul Obs.*), unpublished.
Guentzel-Lingner, U. 1955, *Astron. Nachr.* **282**, 188.
Hopmann, J. 1954, *Mitt. Univ. Stw. Wien*, **7**, 67.
Lamp, E. 1885, *Astron. Nachr.* **112**, 177.
——, 1887, *Astron. Nachr.* **117**, 361.
Rabe, W. 1954, *Circulaire d'information* No. 2.
Wieth-Knudsen, N. 1953, *Lund Obs. Ann.* No. 12.

THE ASTRONOMICAL JOURNAL VOLUME 65, NUMBER 3 APRIL, 19

Study of a Dark Cloud of the Great Rift in Cygnus

ANNE CLINTON PYNE[*]
Whitin Observatory, Wellesley College, Wellesley, Massachusetts

(Received Nov. 27, 1959)

The presence of a dark cloud at 1580 parsecs with an absorption of $6^{m}.0$ is indicated by star counts in a region near the Fish and Platter nebula in Cygnus. The star density in this region appears to be essentially the same as that in the vicinity of the sun.

IN this study, stars were counted on the blue print ($\alpha=19^{h}56^{m}$, $\delta=+30°$) of the National Geographic Sky Atlas bordering the Great Rift in Cygnus, south of the Fish and Platter nebula.

Of the six regions selected for star counts three show strong obscuration while the other three seem to be free of any dark nebulosity. The clear regions lie nearly on the galactic equator. The obscured regions lie in the Great Rift with galactic latitude close to $b^{\mathrm{II}}=-2°$. Each area was a square with sides $22'.7$. The coordinates of the centers are listed in Table I.

TABLE I. Galactic coordinates.

Obscured regions	l^{II}	b^{II}	Clear regions	l^{II}	b^{II}
1	$65^{\circ}.8$	$-2^{\circ}.7$	4	$65^{\circ}.1$	$+0^{\circ}.8$
2	67.8	−2.6	5	66.8	+0.4
3	65.4	−1.0	6	67.3	+0.5

The magnitudes were established for $m=8$ through $m=19$ by direct comparison (flyspanker method) with stars in Selected Area 64 which lies at the center of the Sky Survey print. In each region the value of $N(m)$ for successive magnitudes was counted. $N(m)$ is the number of stars of magnitude m or brighter, where m includes all stars between $m-\frac{1}{2}$ and $m+\frac{1}{2}$. From this the values of $A(m)$ given in Table II were determined.

Counts from the three clear regions were combined to provide a standard with which each obscured region was then compared. In Fig. 1 the logarithms of the counted numbers of stars per square degree of each magnitude are plotted. The length of the vertical line represents the square root of the counted number.

A basic (m, $\log\pi$) table for $b=0°$ from Bok and MacRae (1941) was modified to represent the clear regions. A smooth absorption of $0^{m}.5$/kpc was introduced to account for the general haze along this part of the galactic plane. A marked divergence between th theoretical curve and the counted numbers of stars wa found at magnitude fainter than $m=15$, which coul not be accounted for by any reasonable variation c the star density. Stebbins, Whitford, and Johnso (1950) found systematic errors amounting to as muc as 0.6 mag beyond $m_{\mathrm{pg}}=17$ in three selected areas c the Mt. Wilson Catalog which made these stars appea too bright. If this effect were also true for Selecte Area 64 most of the above disagreement might b accounted for. The clear regions, however, could b well represented by a density function varying onl slightly from that in the vicinity of the sun and th addition of an obscuration of 2 mag at 4000 parsecs This is shown by the curve in Fig. 1a.

Because the obscured regions have their centers clos to $b=-2°$, the basic (m, $\log\pi$) table for this latitud from Bok and MacRae (1941) was used as the startin point in this part of the analysis. To it was added th density and absorption found for the clear areas. I

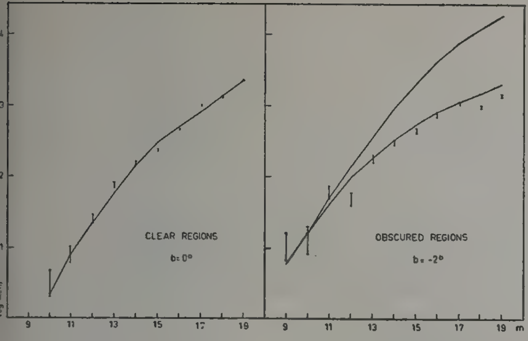
TABLE II. Star counts in Cygnus.

	$\log A(m)$,[*] region					Residuals (observed—computed), margin				
m	1	2	3	4–6 (mean)	1–3 (mean)	1	2	3	4–6	1–3
9	1.32	1.15				−0.67	−0.40			
10	1.32	1.15	0.85	1.48	1.16	−0.24	+0.06	+0.36	+0.01	+0.0
11	1.89	1.85	1.56	1.90	1.79	−0.41	−0.21	+0.08	−0.06	−0.0
12	1.45	1.69	1.85	2.40	1.69	+0.26	+0.31	+0.10	−0.11	+0.2
13	2.28	2.25	2.21	2.85	2.25	−0.33	−0.01	+0.03	−0.15	−0.0
14	2.31	2.45	2.59	3.18	2.47	−0.16	+0.08	−0.06	−0.04	0.0
15	2.39	2.52	2.85	3.36	2.63	−0.10	+0.23	−0.08	+0.11	+0.0
16	2.50	2.84	3.04	3.65	2.84	−0.03	+0.05	−0.10	+0.02	+0.0
17	2.72	2.99	3.18	3.99	3.02	−0.04	+0.03	−0.02	−0.08	0.0
18	2.65	2.93	3.18	4.09	2.97	+0.31	+0.21	+0.18	+0.04	+0.1
19	2.97	2.78	3.39	4.35	3.12	+0.30	+0.49	+0.22	0.00	+0.1

[*] $\log A(m)$ refers to the number of stars per square degree, not the counted number. Separate analyses have been carried out for each of the observe regions and for the mean of the combined regions.

---

[*] Now at University of Michigan Observatory, Ann Arbor, Michigan.



(a) (b)

FIG. 1. (a) Star counts in clear regions. The vertical lines oresent the counted numbers. The curve is the closest approxiation to the counts; it includes a smooth absorption of $0^{m}5$/kpc, ry slight variation in the density function, and an added obration of $2^{m}0$ at 4000 parsecs. Taking the density in the vicinity the sun to be 1.0, the slight variation appears near 1500 parsecs ere the density varied smoothly to 1.2, falling back to 1.0 ar 2500 parsecs. (b) Star counts in the combined obscured gions. The counts along the Rift are plotted on the same scale Fig. 1a. The upper curve represents a clear region at $b^{II}=-2°$ th the addition of the same absorptions and density function for the clear regions of Fig. 1a. The lower curve represents the me region with an additional cloud absorbing $6^{m}0$ at 1580 rsecs.

ldition, all three of the obscured regions were found to ntain dark clouds at approximately the same distance. he results are given in Table III.

TABLE III.

scured region	Distance of cloud (parsecs)	Absorption
1	between 1000 and 1580	$5^{m}2$
2	1580	6.0
3	1580	3.4

Figure 2 shows where other work has been done in is area. Franklin (1955) working directly on the Fish id Platter nebula finds a cloud at approximately the me distance as the one studied here. His region, owever, shows a much lower absorption (which is pparent on the Sky Survey print). Bok and Warwick 957) find obscuring material at somewhat closer stances on the other side of the galactic equator, near e Veil nebula. All of the work in this general area ems to be in good agreement.

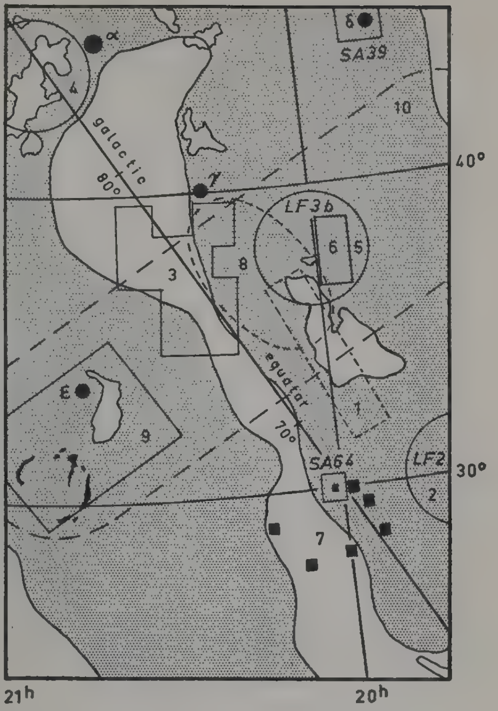


FIG. 2. Main features of the dark nebulosity along the Great Rift in Cygnus, with outlines of regions that have been studied as indicated in the key.

Key:
1. Franklin 1955, *Astron. J.* **60**, 351.
2. McCuskey and Seyfert 1947, *Astrophys. J.* **106**, 1.
3. Miller 1937, *H. A.*, **105**, 297.
4. Muller and Hufnagel, 1949, *Z. Astrophys.* **9**, 331.
5. Nassau and MacRae 1949, *Astrophys. J.* **110**, 40.
6. Nassau and VanAlbada 1949, *Astrophys. J.* **109**, 391.
7. Pyne, present study.
8. Roman 1951, *Astrophys. J.* **114**, 492.
9. Bok and Warwick 1957, *Astron. J.* **62**, 323.
10. Shalen 1928, *Ups. Medd.* **37**, 4.

ACKNOWLEDGMENTS

I wish to thank Mr. Owen Gingerich for his continued help and patience and Dr. Sarah Hill at whose suggestion this project was undertaken. Figure 2 was prepared by Mr. Gingerich.

REFERENCES

Bok, B. J. and MacRae, D. A. 1941, *Annals of the New York Academy of Science* **42**, 113.
Bok, B. J. and Warwick, C. **1957**, *Astron. J.* **62**, 323.
Franklin, F. A. 1955, *Astron. J.* **60**, 351.
Stebbins, J., Whitford, A. E., Johnson, H. C. 1950. *Astrophys. J.* **112**, 969.

THE ASTRONOMICAL JOURNAL VOLUME 65, NUMBER 3 APRIL, 19

Measures of 241 Double Stars*

CHARLES E. WORLEY
Lick Observatory, Mount Hamilton, California

(Received January 13, 1960)

1062 measures of 241 double stars, made with the 12-in. refractor of Lick Observatory, are presented.

THE observations of double stars contained in Table I form the third series of measures made by the writer with the 12-in. refractor of the Lick Observatory. Table I contains 1062 measures of 241 pairs, and includes 45 pairs measured in more than one season. The arrangement of the material in the table is self-explanatory. The tabulated differences of magnitude, Δm, are visual estimates. Residuals from orbits have been computed for all pairs whenever ephemerides were conveniently available. In all cases, the orbit giving the smallest residuals has been quoted. A few notes are included at the end of the table.

The observing program is a continuation of that presented in the two earlier series (Worley 1956, 1957). It is made up primarily of pairs having separations of less than two seconds and showing orbital motion. Special emphasis is placed on pairs for which orbits have been computed. A few pairs have been observed more frequently than strictly necessary in order to provide material for a future evaluation of the accidental and systematic errors of this and the preceding series of measures.

The Clark micrometer, which has a screw value 14″.059 per revolution, was used for all measures. Th power employed was nearly always 625, but a fe measures were made with powers of 545 and 1000. measure ordinarily consisted of four or five settings fo position angle (wires set parallel to the line joining th stars), followed by two or three settings for doubl distance. Experience has shown that making more tha three settings for the distance simply tends to repea the first few settings and is not an appreciable con tribution to the accuracy. Instead, the average numbe of nights of measurement for each pair has been in creased to 3.5, in contrast to the 3.1 and 3.0 of serie one and two, respectively.

I am indebted to Dr. C. D. Shane and Dr. A. E Whitford for their support of this program during th three years prior to July 1, 1959. The work done sinc that date has been supported by the Office of Nava Research.

REFERENCES

Worley, C. E. 1956, *Astron. J.* **61**, 162.
——. 1957, *Astron. J.* **62**, 153.

---

* Lick Observatory Bulletin, No. 564.

TABLE I. Measures of 241 double stars.

DS/BD	Name	Date	θ	A.D.	ρ	A.D.	Nights	Δm	Residuals from orbits
			°	°	″	″			
9	β 281	1959.74	179.0	0.7	1.62	0.05	4	2.1	
10	A 800	57.64	290.5	0.6	1.62	0.07	3	0.0	
60	Σ 3060	59.63	126.8	0.3	3.40	0.03	4	0.1	
61	Σ 3062	57.64	233.5	0.8	1.22	0.03	3	1.0	
		58.94	235.3	1.3	1.34	0.02	3	1.1	Baize, 1957: -2.°2, +0.″09.
61	Σ 3062	59.72	239.5	0.6	1.36	0.02	5	1.0	Baize, 1957: -0.°2, +0.″10.
102	Σ 2	56.90	36.0	0.7	0.45	0.01	3	0.2	Pensado, 1954: +1.°6, +0.″02.
148 *	β 1026	57.83	336:		0.35e		4	0.8	Baize, 1955: +10.°6, 0.″00.
207	Σ 13	59.80	64.3	0.5	0.94	0.01	3	0.2	
382	A 1504AB	57.70	27.4	0.8	0.52	0.06	4	0.0	
399	A 909AB	57.76	40.1	0.9	1.06	0.10	3	0.5	
434	λ Cas	59.78	175.1	1.4	0.63	0.04	4	0.1	
588	OΣ 18	59.66	183.0	0.7	1.53	0.06	4	1.8	Baize, 1958: -2.°3, +0.″14.
650	Hu 413	57.69	262.8	0.7	0.63	0.02	3	1.2	
673	β 495	59.68	191.0	2.5	0.45	0.04	5	0.1	
684	β 232	59.76	224.9	1.9	0.69	0.03	4	0.4	van Biesbroeck, 1954: -2.°7, +0.″11.
746	OΣ 20	59.67	252.0	1.6	0.56	0.02	5	1.1	Rabe, 1958: +6.°6, +0.″13.
755	Σ 73	57.65	175.9	1.3	0.69	0.03	4	0.5	Muller, 1957: -0.°3, -0.″01.
		58.95	179.3	0.6	0.77	0.05	4	0.7	-0.8, +0.07.
		59.62	182.0	0.8	0.74	0.03	5	0.4	-0.2, +0.05.
940	φ And	56.92	160.1	0.7	0.45	0.03	4		
		59.76	155.4	0.8	0.49	0.03	4	2.0	Baize, 1958: +0.°6, +0.″07.
1200	A 942	58.36	42.3	1.7	1.74	0.04	2	0.6	
1223	Howe 4	57.85	353.3	2.0	0.67	0.02	3	0.3	van den Bos, 1953: +5.°1, +0.″15.
1300	β 5	57.69	285.1	1.4	0.98	0.10	4	1.8	
1339	Σ 147	57.98	90.6	0.4	2.15	0.04	2		
1359	β 870	56.92	20.2	1.5	1.05	0.04	4	1.2	
1438	Σ 162	56.83	208.1	1.7	1.93	0.05	3		
1538	Σ 186	57.00	49.9	0.5	1.42	0.05	3	0.0	Cid, 1952: +1.°8, -0.″06.
1598	β 513	57.70	350.2	1.6	0.53	0.04	4	2.5	Muller, 1949: -5.°7, +0.″07.
1631	Σ 208	57.36	274.2	3.4	0.53	0.04	7	2.2	
1709	Σ 228	56.93	241.2	0.1	0.81	0.01	3	0.4	Gonzalez-Aboin, 1953: -0.°5, +0.″09.
		58.94	245.4	1.5	0.78	0.08	3	0.9	+0.2, +0.02.
1778*	o Cet	57.58	124.4	0.4	0.67	0.05	2		
1786*	Σ 248	56.93	127.0	1.9	0.78	0.01	3	0.1	
1786	Σ 248	59.43	122.1	2.0	0.72	0.03	4	0.1	
1° 431	Kuiper	57.70	26.6	0.5	0.48	0.02	4	0.3	
1860	ι Cas AB	58.95	236.1	0.4	2.39	0.05	4	2.0	Rabe, 1954: -4.°0, +0.″07.
	AC	58.94	113.7	0.5	7.18	0.08	3	3.0	
2117	β 9	59.30	184.3	0.8	1.49	0.07	4	1.8	
2236	A 2413	57.75	24.2	0.9	0.53	0.01	4	0.1	Muller, 1951: -5.°4, +0.″07.
2377	OΣ 50	56.90	179.6	0.7	1.21	0.06	3		
2416*	Σ 367	59.79	155.7	1.0	0.99	0.05	4	0.0	Dommanget, 1954: +3.°4, +0.″22.
2436	OΣ 52	59.58	84.0	0.9	0.53	0.01	4	0.6	
2446	OΣ 53	56.93	277.9	1.5	0.74	0.04	3	0.6	Rabe, 1948: -2.°1, -0.″02.
2459	AC 2	57.20	237.9	0.6	1.06	0.03	3	2.3	
2612	Σ 400	57.75	243.9	0.8	0.58	0.04	4	0.8	Baize, 1952: -0.°4, -0.″02.
2616	Σ 412	56.94	19.2	1.4	0.48	0.07	2		Luyten, 1934I: -0.°9, 0.″00.
2628	β 533	57.54	44.2	1.2	1.06	0.04	4	0.2	
2873	β 263	57.95	90.6	3.3	0.88	0.06	2	0.2	
2959	Σ 483	57.76	118.7	1.5	0.78	0.06	3	1.6	
2963	Σ 460	56.94	92.2	0.0	0.94	0.00	2		
3082	OΣ 77	57.95	258.2	1.0	0.81	0.01	4		Muller, 1956: -1.°2, +0.″02.
3093*	Σ 518	57.88	347.8	0.5	7.74	0.09	4	2.0	van den Bos, 1926: 0.°0, +0.″49.
3114	Σ 520	57.51	147.3	5.0	0.37	0.07	5		

TABLE I. (Continued)

ADS/BD	Name	Date	θ	A.D.	ρ	A.D.	Nights	Δm	Residuals from orbits
3153	A 1836	1956.99	193°5	0°2	1″41	0″09	3	0.2	
3169	OΣ 82	57.88	14.0	0.8	1.29	0.10	4	1.8	Baize, 1957: -2°9, +0″02.
3174	Σ 535	57.85	304.9	0.3	1.42	0.05	3	1.6	
3391	A 1013	57.04	349.0	1.6	0.36	0.02	2		
3483*	β 552	57.27	314.3	1.7	1.00	0.08	6	1.8	Baize, 1953: +7°9, -0″02.
+73°265	OΣ 89	57.63	296.0	1.6	0.62	0.08	4	1.7	
3593	Σ 619	56.99	139.9	0.1	4.19	0.04	3	0.2	
3596	OΣ 93	56.96	254.5	1.4	0.67	0.04	3	1.2	
3672	OΣ 95	56.93	311.8	0.2	0.95	0.01	3	0.2	
3711	OΣ 98	56.90	77.0	0.4	0.85	0.02	4	0.8	Baize, 1953: -1°1, -0″03.
3711	OΣ 98	57.85	76.6	0.6	0.85	0.02	3	0.7	Baize, 1953: +0°4, -0″02.
		59.06	73.8	0.8	0.89	0.05	3	0.5	0.0, +0.04.
3777	A 51	57.04	100.2	1.2	1.58	0.03	2	0.2	
4082	A 1720	56.90	270.5	0.4	2.03	0.07	3	0.6	
4305	Σ 778	56.99	183.2		3.10		1	0.9	
4317	A 1040	57.30	107.8	0.6	0.90	0.02	3	0.6	
4371	β 560	56.93	137.1	0.1	1.32	0.02	3		
4472	β 1053	57.64	348.7	1.4	1.41	0.02	4	1.8	
4617	μ Ori	57.94	23:		0.29e		4		Alden, 1942: +3°8, +0″03.
4971	A 2667	56.94	119.7	2.1	0.48	0.03	4	0.8	Muller, 1951: +1°9, -0.03.
5016	A 2356	56.96	80.0	1.4	0.90	0.01	3	0.1	
5234	OΣ 149	56.97	10.1	2.4	0.60	0.02	4	2.0	Rabe, 1948: -4°5, +0″08.
5400	Σ 948AB	59.04	88.3	0.1	1.73	0.01	2	0.3	Brosche, 1957: -1°0, +0.05.
	AC	59.04	308.8	0.3	8.72	0.04	2	1.2	
5519	A 1956	57.91	241.9		0.59		1		
5553	OΣ 160	56.93	181.2	0.6	1.47	0.07	3	2.8	
5586	OΣ 159	56.99	33.9	0.6	0.94	0.02	3	0.7	
5704	A 1575	57.60	280.5	0.5	0.82	0.02	3		
5841*	J 703	57.31	302.1	0.7	5.74	0.21	3	0.1	
6038	Σ 1081	56.93	231.6	0.4	1.70	0.03	3	0.6	
6085	A 2467	57.60	54.4	0.2	1.42	0.09	3	1.0	
6175	α Gem	56.95	174.0	0.3	2.33	0.02	5		
		57.95	172.8	1.5	2.30	0.05	5		Rabe, 1958: -0°4, -0″03.
		59.06	171.0	0.3	2.23	0.01	3	1.0	+0.8, -0.03.
6263	Σ 1126	56.96	156.4	0.4	1.10	0.00	3		
6295	A 2877	56.94	62.6	1.1	0.71	0.02	4	0.3	
6347	Ho 247	57.45	188.3	1.0	0.40	0.05	2		
6365	Ho 36	57.31	310.3	3.1	0.63	0.02	3	0.5	
6454	Σ 1157	56.96	220.9	0.3	0.97	0.02	3	0.0	
6483	OΣ 185	56.99	19.6	0.4	0.45	0.01	2		van den Bos, 1949: -0°1, +0″04.
6650	ζ CncAB	56.94	15.0	1.0	1.14	0.02	4		Gasteyer, 1954: +0°4, -0″02.
		57.96	10.1	1.0	1.15	0.04	4		-1.2, -0.01.
		59.06	7.1	0.8	1.16	0.06	3	0.3	-0.5, 0.00.
	AC	59.10	84.4	0.2	5.94	0.05	2		-0.5, -0.12.
6762	Σ 1216	56.96	255.9	0.4	0.55	0.01	3		Ekenberg, 1945: -2.2, +0.02.
7054	A 1584	57.10	107.2	0.5	0.70	0.01	2		
+42°1956*	Kuiper	57.06	37.0	0.7	0.61	0.02	3		Baize, 1955: -1°8, -0″03.
		57.96	29.0	0.3	0.74	0.03	3	1.2	-1.4, +0.06.
		59.10	17.0		0.83		1		-3.4, +0.13.
7081	AG	57.66	204.4	0.4	1.55	0.02	3	0.2	
7203	Σ 1306	57.06	26.4	0.5	2.30	0.02	3		Baize, 1948: +4°3, -0″07.
7341	A 2477	57.68	296.6	1.9	0.41	0.02	3		
7499	A 2761	56.99	247.4	0.6	0.84	0.04	4	0.2	
7545	φ UMa	58.54	19.4	1.6	0.50	0.02	2	0.5	Wierzbinski, 1957: -2.3, +0″11.
7550	A 2561	57.37	309.8	1.8	0.54	0.03	3		

TABLE I. (Continued)

/BD	Name	Date	θ	A.D.	ρ	A.D.	Nights	Δm	Residuals from orbits
7704	OΣ 215	1957.02	188°1	0°6	1″33	0″01	3	0.1	Abascal, 1957: 0°0, -0″01.
7758	Σ 1429	58.52	204.6	2.8	0.66	0.04	2	0.0	
7864	Σ 1457	58.54	325.8	0.3	1.80	0.03	2	1.2	
7896	A 2768	57.53	207.3	1.4	0.49	0.04	2		
8119	ξ UMa	57.26	165.2	1.4	1.94	0.12	4		Rakowiecki, 1938: +0°4, +0″04.
8119	ξ UMa	58.08	161.8	1.5	2.03	0.08	4	0.6	Rakowiecki, 1938: +1°4, +0″06.
		59.39	155.3	0.1	2.13	0.06	4	0.2	+0.7, +0.04.
8148	ι Leo	57.53	221.8	0.7	1.08	0.05	2		Baize, 1952: +1°5, +0.06.
		58.46	215.4	1.0	1.06	0.05	3	2.8	-1.8, +0.02.
8197	OΣ 235	57.51	70.5	1.0	1.02	0.02	2		Hable, 1954: +3°9, -0.03.
8197	OΣ 235	59.39	72.0	0.8	1.07	0.05	4	1.7	Hable, 1954: +1°8, +0″04.
8539	Σ 1639	57.64	331.1	1.1	1.36	0.04	3	0.8	Schrutka-Rechtenstamm, 1950: +1°5, +0″10.
8596	Ho 537	56.99	175.5		1.74		1		
8708	OΣ 256	58.00	90.8	0.8	0.88	0.05	3	0.2	
8739	β 1082	58.46	6.8	1.1	1.16	0.12	3	3.3	Baize, 1948: +6°2, +0″14.
8772	β 799	59.50	260.0	0.6	1.19	0.03	4	1.6	
8804	Σ 1728	57.37	192.4	0.7	0.73	0.04	4	0.0	Pavel, 1949: +0°5, +0″11.
		58.46	191.2	0.2	0.75	0.04	3	0.1	-0.7, +0.12.
		59.49	192.7	0.6	0.69	0.02	4	0.0	+0.8, +0.09.
8841*	β 800	59.49	107.6	0.2	6.36	0.10	4	3.5	
8862	Hu 644	57.34	92.0	0.7	1.23	0.04	3		Baize, 1949: -1°8, +0″01.
		59.52	91.5	1.1	1.29	0.04	4	1.1	-0.5, +0.03.
8901	A 1609AB	58.46	16.1	1.3	0.55	0.01	2	0.0	Baize, 1957: +3.9, +0.06.
8974	Σ 1768	59.49	109.6	0.7	1.77	0.08	4	1.6	Jackson, 1921: +2°6, +0″10.
9019	Σ 1781	58.46	343.1	2.3	0.66	0.02	3	0.5	Arend, 1953: +3°9, +0″09.
9019	Σ 1781	59.52	347.2	0.7	0.68	0.03	3	0.4	Arend, 1953: +5°5, +0″13.
9031	Σ 1785	59.51	141.9	0.5	2.93	0.08	4	0.2	Strand, 1955: -0°8, +0″03.
9071	A 1614	58.46	146.6	1.7	1.12	0.04	3	0.1	Muller, 1957: +2°1, +0″21.
9182	Σ 1819	57.51	289.8	0.3	0.97	0.03	2		Hopmann, 1945: -0°5, -0″01.
	Σ 1819	59.52	285.8	0.4	1.01	0.05	4	0.1	-1.0, +0.03.
9229	Σ 1834	59.53	99.6	0.6	1.29	0.04	3	0.2	van den Bos, 1938: -2°6, +0″27.
9324	A 347	58.46	303.1	0.9	0.50	0.04	3	0.1	Guntzel-Lingner, 1956: +0°8, +0″04.
9352	Hu 575	58.46	158.1	1.8	0.55	0.06	3	0.8	Muller, 1952: +0°4, -0″02.
9392	Σ 1883	58.46	101.5	2.2	0.36	0.04	3	0.0	Pensado, 1951: -1°4, -0″06.
9418	OΣ 287	59.50	160.5	0.6	1.20	0.05	4	0.1	
9425	OΣ 288	58.46	177.4	0.1	1.67	0.01	3	0.6	Heintz, 1956: -0°4, +0″09.
9494	Σ 1909	58.46	262.0	1.0	1.15	0.06	3	0.7	Gennaro, 1940: -3°8, +0″12.
		59.50	265.2	0.5	1.03	0.06	4	0.5	-2.8, +0.08.
9578	Σ 1932	58.46	231.9	0.4	0.91	0.05	3	0.1	van Biesbroeck, 1954: -2°1, -0″02.
9617	η CrB	58.46	64.7	1.1	0.79	0.02	3	0.2	Danjon, 1938: -0°5, +0″08.
9617	η CrB	59.51	71.3	0.4	0.73	0.02	4	0.4	Danjon, 1938: -0°7, +0″07.
9626	μ BooBC	58.46	23.7	1.0	1.92	0.06	3	0.4	Baize, 1952: -0°9, -0″05.
9639	OΣ 296	59.55	286.5	0.5	1.94	0.08	4	1.5	
9716	OΣ 298	58.46	182.6	0.9	1.26	0.05	3	0.1	Stephens, 1939: -0°4, -0″02.
9769	Σ 1989	59.53	35.7	1.5	0.64	0.03	4	0.7	Wilson, 1935: +3°7, 0″00.
9880	OΣ 303	59.52	162.3	0.9	1.26	0.08	4	0.3	
9909	ξ ScoAB	57.52	334.5	0.8	0.68	0.06	3		Baize, 1942: +0°2, +0″04.
		58.46	339.8	0.4	0.76	0.03	3	0.0	+1.1, +0.05.
		59.51	343.6	0.4	0.78	0.05	4	0.2	+0.8, 0.00.
	AB-C	58.46	53.3	0.5	7.55	0.04	3		
9925	β 812	57.55	110.3	1.1	0.68	0.02	3	0.6	
9982	Σ 2026	57.52	29.6	0.5	2.20	0.11	3		Woolley+Symms, 1937: -2°5, 0″00.
		59.51	29.0	0.6	2.40	0.07	4	0.3	-2.4, +0.13.
0075	Σ 2052	57.52	165.2	0.3	0.85	0.03	3		Siegrist, 1952: +0°2, +0″04.
		59.53	161.2	0.2	0.99	0.04	5	0.3	+0.4, +0.13.

TABLE I. (Continued)

ADS/BD	Name	Date	θ	A.D.	ρ	A.D.	Nights	Δm	Residuals from orbits
10087	λ Oph	1957.55	336.°9	0.°4	0.″82	0.″06	4	0.5	Rabe, 1948: +5.°5, -0.″01.
		58.46	340.5	0.2	0.91	0.01	3	0.8	+6.8, +0.06.
		59.51	341.9	0.8	0.90	0.03	4	0.8	+5.8, +0.03.
10093	Σ 2059	59.65	198.6	1.0	0.91	0.02	3	0.2	
10157	ζ Her	57.52	73.5	0.3	1.44	0.07	3		Baize, 1949: -0.°3, -0.″14.
10157	ζ Her	58.46	69.3	0.5	1.62	0.03	3	1.5	Baize, 1949: -0.°1, +0.″06.
		59.51	65.9	0.8	1.54	0.02	4	1.8	+1.5, +0.02.
10188	D 15	58.46	159.2	0.4	1.44	0.02	3	0.0	van Biesbroeck, 1927: +1.°9, +0.″26.
10230	OΣ 315	57.69	130.2	2.2	0.52	0.03	4	2.1	
		58.87	131.3	2.1	0.49	0.05	5	2.2	
10235	Σ 2107	57.64	72.2	1.0	1.19	0.05	3	1.3	Rabe, 1927: -2.°0, -0.″03.
10279	Σ 2118	57.58	71.2	0.3	1.06	0.01	3	0.1	Giannuzzi, 1956: +0.°4, +0.″02.
10295	β 1298	57.70	115.0	0.2	0.44	0.02	2	0.5	
		59.56	116.0	1.0	0.52	0.04	6	0.5	
10345	μ Dra	58.46	74.8	0.8	2.09	0.10	3	0.0	von Bezold, 1938: -1.°6, -0.″07.
10345	μ Dra	59.55	74.6	0.6	2.16	0.06	5	0.0	+0.°3, +0.″01.
10398	OΣ 325	59.61	227.2	1.8	0.66	0.04	4	1.9	
+45°2505	Kuiper	57.70	245.2	0.1	1.08	0.06	2	0.4	van Biesbroeck, 1949: -0.°7, -0.″02.
		58.60	238.8	1.2	1.12	0.05	4	0.8	-4.2, 0.00.
		59.52	232.7	0.9	1.23	0.07	4	0.8	-0.8, +0.12.
10429	A 2984	59.04	345.6	1.4	1.07	0.13	5	2.7	
10517	β 46	59.64	202.4	1.1	2.13	0.10	4	3.2	
10585	A 351	59.49	61.3		0.55		1	0.4	Baize, 1955: +0.°8, +0.″04.
10660	β 962	59.63	144.8	0.8	1.31	0.07	6	3.1	Hall, 1949: +0.°3, +0.″13.
10675	AG 210	59.58	181.1	0.6	4.26	0.04	4	0.3	
10769	Σ 2205	57.69	327.4	0.2	1.78	0.06	3	0.1	
10786	μ HerBC	57.66	150.3	2.2	0.52	0.05	4	1.0	Silbernagel, 1928: -3.°8, +0.″02.
		58.55	171.0	2.3	0.62	0.06	3	1.0	-2.2, +0.09.
		59.56	190.9	1.9	0.68	0.06	3	1.0	-1.1, +0.09.
10795	Σ 2215	57.62	277.1	1.9	0.66	0.02	3	1.6	
10912	Σ 2244	58.64	284.5	1.0	0.32	0.02	3	0.0	
10990*	β 1125	59.53	63.6	0.2	0.94	0.04	3	4.2	
11001	Σ 2267	59.69	253.4	1.8	0.90	0.04	4	0.0	
11005	τ Oph	58.46	271.6	0.5	1.98	0.05	3	0.4	Hertzsprung, 1942: +0.°1, +0.″01.
11010	β 1127	59.69	89.6	0.4	1.08	0.05	4	1.7	
11046	Σ 2272	57.55	96.0	0.4	4.78	0.06	4		Strand, 1952: +0.°4, -0.″06.
		59.50	92.3	0.4	4.52	0.08	5		+0.1, +0.06.
11077	AC 15	57.57	326.6	1.6	1.30	0.06	4	3.2	Ames+Pratt, 1958: +2.°5, -0.″14.
		58.60	330.2	1.9	1.38	0.08	4	3.5	+4.1, -0.10.
		59.56	332.3	0.4	1.54	0.08	7	3.5	+4.3, +0.04.
11111	Σ 2281	57.64	33.8	0.6	0.39	0.04	3	1.2	van den Bos, 1937: -2.°9, -0.″09.
11186	Σ 2294	57.52	95.4	0.5	0.98	0.09	3		Wilson, 1935: +0.°1, +0.″04.
11257	Schj 16	57.55	203.3	0.5	2.86	0.10	3	1.2	
11260	Hu 197	59.70	211.4	1.7	0.52	0.03	3	1.0	Baize, 1956II: -1.°7, +0.″06.
11262	Σ 2303	59.63	236.0	0.8	2.12	0.05	3	2.3	
11334*	Σ 2315	58.64	141.6	0.5	0.63	0.05	3	1.2	Voronov, 1933: +6.°8, +0.″29.
11344AB	Hu 66	58.64	268.3	1.2	0.46	0.05	3	0.2	
AC	OΣ 351	58.64	19.0	0.6	0.89	0.00	3	0.5	
11479	OΣ 359	59.50	16.9	1.2	0.58	0.03	4	0.2	
11483	OΣ 358	59.50	170.8	0.4	1.87	0.04	4	0.1	Heintz, 1955: 0.°0, +0.″08.
11568	Σ 2384	59.72	309.4	1.9	0.93	0.02	4	0.8	Baize, 1950: -0.°6, -0.″09.
11844	A 2193	59.62	348.7	0.6	0.73	0.08	4	0.0	
11867	Hu 676	59.64	76.8	0.7	1.39	0.18	4	3.0	
11869	Σ 2422	57.52	80.6	0.5	0.79	0.05	3		
11871*	β 648	57.62	233.2	1.1	1.37	0.11	4	1.8	Schrutka-Rechtenstamm, 1940:+0.°1, +0.″

TABLE I. (Continued)

BD	Name	Date	θ	A.D.	ρ	A.D.	Nights	Δm	Residuals from orbits
					″	″			
871	β 648	1958.64	229°3	0°3	1.40	0.04	3	1.5	Schrutka-Rechenstamm, 1940: -1°4, +0″16.
		59.59	228.8	1.8	1.39	0.08	7	2.3	+0.4, +0.17.
1897	Σ 2438	59.60	6.2	0.8	0.89	0.03	5	0.3	Pensado, 1952: +0°2, +0″09.
1956	Σ 2437	57.58	36.4	0.5	0.73	0.04	3	0.2	
2040	Σ 2454	59.76	274.5	1.3	1.22	0.04	3	1.3	
2140	A 151	57.71	144.2	0.4	0.64	0.05	2	1.5	
2147	β 1204	59.74	184.3	2.3	0.46	0.02	3	0.8	
2172	Σ 2482	59.76	344.3	0.4	1.87	0.05	4	1.0	
2201	Σ 2484	59.70	234.0	0.6	2.47	0.06	3	1.4	
2228	Σ 2488	59.76	342.8	0.9	1.73	0.06	4	1.0	
2261	A 1392	59.80	81.8	0.3	0.51	0.02	4	0.4	
2447	Σ 2525	58.52	295.8	0.6	1.68	0.03	6	0.0	Finsen, 1937: +1°1, +0″21.
2623	OΣ 375	57.52	166.6	0.6	0.69	0.03	3		
3767	van de Kamp	59.49	132.4		3.73		1		
2880	δ Cyg	57.60	246.7	0.3	2.21	0.02	3	3.2	Rabe, 1954: -0°3, +0″09.
2889	Σ 2576	57.59	32.9	0.3	1.14	0.04	4	0.0	Cester, 1949: -0°8, -0″05.
		58.64	29.3	0.8	1.23	0.07	3	0.1	-1.5, -0.02.
		59.52	27.7	0.4	1.34	0.06	4	0.0	-0.8, +0.05.
2972	OΣ 387	59.62	207.4	0.1	0.59	0.01	4	0.8	Muller, 1952: +5.0, +0.14.
3028	β 148	59.67	271.9	1.7	0.69	0.03	4	0.6	
3055	Ho 580	59.65	273.1	1.1	0.87	0.09	5	0.1	
3082	Σ 2596	59.73	311.0	0.1	2.22	0.01	3	1.4	
3104	Σ 2597	59.50	82.1	1.5	0.61	0.04	4	1.3	
3178	AC 12	59.64	308.1	0.5	1.40	0.03	4	0.8	
3189	A 1410	59.79	197.7	1.6	1.85	0.07	4	0.7	
3277	OΣ 395	57.52	116.6	0.9	0.95	0.05	3		
		59.68	114.8	0.8	0.96	0.04	4	0.3	
3461	OΣ 400	57.62	290.0	2.4	0.35	0.04	4	0.5	Wierzbinski, 1957: -2°4, +0″07.
3723	OΣ 406	57.60	124.2	0.5	0.55	0.03	3	1.0	Wierzbinski, 1957: +1.5, +0.03.
		59.75	125.8	0.6	0.61	0.04	4	1.2	+3.6, +0.08.
13857	A 1429	59.69	190.4	2.5	0.82	0.03	4	0.9	
14056	A 2793	59.40	214.2	1.6	0.92	0.08	4	0.5	
14073	β Del	57.57	34.6	1.0	0.37	0.02	3	0.8	Finsen, 1938: -2°6, +0″07.
		58.64	49.6	0.6	0.24	0.01	3		-9.3, +0.01.
°4510	Kuiper	57.64	135.0	1.6	0.64	0.05	5	1.4	Baize, 1957: +1°3, +0″06.
°4510	Kuiper	59.29	135.6	2.0	0.56	0.02	6	1.0	Baize, 1957: -2.5, +0.07.
14238	β 64	57.70	153.6	0.9	0.51	0.06	4	0.2	Wierzbinski, 1949: -1°3, +0″05.
		59.60	154.6	0.8	0.62	0.03	4	0.1	-1.8, +0.14.
14243	A 876	59.42	67.6	0.5	0.74	0.02	3	0.3	
14296	λ Cyg	57.64	27.9	1.4	0.85	0.08	4	2.0	Rabe, 1948: +1°2, +0″06.
14355	β 67	59.76	303.2	1.6	1.56	0.09	4	2.2	
14360	Σ 2729	59.62	2.4	0.6	1.16	0.04	4	0.7	Baize, 1943: -0°1, +0″11.
14421	OΣ 418	59.65	286.9	0.7	1.24	0.06	4	0.0	
14499	ϵ Equ	57.70	288.1	0.6	0.82	0.02	3	-	van den Bos, 1933: +1°3, -0″07.
		59.49	287.3	0.1	0.93	0.06	4	0.2	+0.6, +0.01.
14573	Σ 2744	59.54	137.6	0.5	1.50	0.05	4	0.5	
14783	HI 48	59.70	249.1	0.5	0.97	0.02	4	0.2	Baize, 1950: +0°3, +0″05.
14784	Σ 2783	59.70	15.0	0.7	0.94	0.06	4	0.1	
14787	τ Cyg	57.62	252.8	0.8	0.63	0.01	4	2.8	van Biesbroeck, 1940: -0°6, -0″08.
		58.74	245.4		0.62		1	3.0	-0.1, -0.10.
14787	τ Cyg	59.71	238.0	2.1	0.77	0.03	5	2.3	-0.6, +0.04.
14839	β 163	57.62	256.3	0.7	0.91	0.03	3	2.2	Baize, 1956: +3°2, +0″02.
		59.55	257.0	1.0	0.95	0.07	5	1.8	+4.3, +0.09.
15176	β 1212	59.41	284.8	2.4	0.57	0.02	5	0.8	Danjon, 1942: -0°7, +0″07.
15270*	μ Cyg	59.45	284.1	0.4	1.50	0.02	5	1.2	Komendantoff, 1936: +6°0, -0″15.

TABLE I. (Continued)

ADS/BD	Name	Date	θ	A.D.	ρ	A.D.	Nights	Δm	Residuals from orbits
15748	A 626	1957.62	272°5	1°7	0″74	0″02	3	0.0	
15902	β 172	57.69	326.4	0.5	0.58	0.03	3	0.2	
15971	ζ Aqr	57.64	267.1	0.7	1.85	0.02	3	0.1	Rabe, 1954: +0°5, -0″17.
		59.46	263.4	1.0	1.90	0.02	6	0.1	-0.1, -0.09.
15972	Kr 60	56.81	86.6	0.9	2.81	0.13	5		Hall, 1952: -1°4, -0″13.
15972	Kr 60	57.62	83.3	1.1	2.71	0.04	4	1.3	-0.8, -0.14.
		58.74	77.2		2.63		1		-0.9, -0.09.
		59.66	73.0	0.4	2.59	0.01	4	1.4	+0.1, -0.02.
15988	Σ 2912	57.57	117.2	0.3	1.06	0.05	3	1.0	van Biesbroeck, 1927: +0°2, +0″02
		59.68	117.6	0.4	1.08	0.02	4	1.2	+0.6, +0.03
16057	Σ 2924	57.71	73.3	1.2	0.55	0.04	3	0.2	Heintz, 1956: +0°4, +0″09.
		59.64	74.2	0.0	0.56	0.01	4	0.1	-0.3, +0.07.
16173	Ho 296	57.64	31.7	1.3	0.36	0.03	3	0.5	Baize, 1957: -1°7, +0″01.
		58.74	25.1		0.28		1		-0.8, -0.01.
16185	Σ 2934	59.67	91.2	1.6	1.09	0.05	4	1.2	
16326	A 632	57.60	183.5	0.6	1.08	0.07	3	0.3	Dommanget, 1953: +2°0, +0″08.
16345	β 382	57.62	175.1	1.1	0.55	0.04	4	2.2	Muller, 1954: +1°0, -0″05.
16428	β 483	59.67	271.5	1.0	0.82	0.09	5	1.3	Guntzel-Lingner, 1956: -2°0, +0″1
16538	π Cep	57.81	305.9	3.6	0.70	0.06	4	2.6	Muller, 1951: +3°2, -0″06.
16561	β 385	57.70	100.2	0.8	0.65	0.03	3		
16649	β 79	59.63	33.4	1.1	1.48	0.03	4	1.5	
16665	β 80	59.45	291.6	1.3	1.39	0.04	6	1.0	Tannenbaum, 1938: +3°7, +0″10.
16726	A 1483	56.92	317.8	2.0	0.74	0.08	4	0.2	
16877	OΣ 500	56.93	346.0	0.3	0.67	0.01	3	0.7	
17006	β 995	56.93	243.8	1.8	0.75	0.04	3	1.9	
17020	OΣ 507	59.80	296.6	0.8	0.76	0.04	4	0.6	
17111	A 2100	57.64	226.6	0.8	0.47	0.04	3	0.8	
17149	Σ 3050	59.72	276.1	0.7	1.52	0.06	4	0.1	Heintz, 1954: +2°5, +0″04.
17175	β 733	57.66	274.0	1.4	0.74	0.06	4	3.8	Hall, 1949: -3°1, -0″06.
17178	Hn 60	56.93	194.7	0.8	0.85	0.03	3	0.3	Dommanget, 1953: -0°5, +0″13.

NOTES TO TABLE I

ADS/BD

148 This pair is very difficult for the 12-in. refractor. Two measures of nearly the same date as the writer's are

1957.61	326°4	0″30	3n	VBS
1957.62	331.0	0.34	3	B.

1778 The difference of magnitude was estimated as 0.6 on 57.550, and 1.8 on 57.624.

1786 This pair is closing in, and will probably become difficult in a few years. Since discovery in 1832, the distance has decreased from 1″6 to 0″7 with a change of only 40° in position angle.

2416 Recent measures seem to indicate a larger distance than predicted by the orbit.

3093 o^2 Eri BC is a difficult pair for the 12-in. refractor. Measures with the 36-in. indicate that the distance measured with the 12-in. is about 0″2 too large. Nevertheless, the deviation from the orbit is appreciable.

3483 Motion in angle seems to be more rapid than predicted.

5841 The writer's individual measures of this faint (estimated magnitudes 11.0–11.1) pair are tabulated below:

1956.99	302°0	5″42
56.99	301.2	5.75
57.91	303.1	6.05

The few measures of this pair since discovery are:

1912.01	331.2	2.41	1 J, 1 Vdk
25.23	316.8	2.15	1 Gcb
49.74	303.8	5.28	4 VBS
52.15	303.9	4.91	2 COU

Several observers reverse the quadrant, although th companion seemed to be in the fourth quadrant t this writer. The pair is probably optical.

+42°1956 This pair has entered in the literature as 10 UMa. is not, however, in Ursa Major, and that designatic should be dropped. (See *Publs. Astron. Soc. Pacifi* 70, 465, 1958.)

8841 Residuals from Eggen's linear formulae are +0° 0″00.

10990 Because of the large difference of magnitude, th pair is very difficult for the 12-in., and on thre additional nights of good seeing could not be meas ured. Measures with the 36-in. give a distance of 0″7

11334 The orbit no longer represents the motion of this pai However, the position angle has changed by onl 140° since discovery, and no reliable orbit can b computed for a long time.

11871 The distance may have been measured too larg although in the same part of the orbit in 1896–189 the better observers measured the distance similarl large.

15270 A new orbit could be computed.

HE ASTRONOMICAL JOURNAL VOLUME 65, NUMBER 3 APRIL, 1960

Observations of Comets and of Icarus (1566)*

H. M. Jeffers and James Gibson
Lick Observatory, Mount Hamilton, California

(Received January 20, 1960)

The paper includes 59 measures of the position of comets, and of one minor planet, Icarus (1566), visible during the period April 1958 to October 1959. Descriptions of appearance and estimates of magnitude are also given.

THE list of observations in this paper continues that given in *Lick Observatory Bulletin* No. 559, ublished in the *Astronomical Journal* **63**, 249 (1958). All the observations are from plates obtained with e Crossley reflector. With each position observations o initials are given; the first indicates the observer, d the second identifies the person who measured and duced the plates. The persons who participated in e work are J. E. Forbes (F), James Gibson (G), and . M. Jeffers (J). We are especially indebted to Mr. orbes, since his work was on a volunteer basis.

As in previous lists, reference star positions for the plates were nearly always obtained from the *Astrographic Catalogue*, preference being given to fainter stars. In a few cases, positions and proper motions were available from more modern catalogues. The magnitudes were estimated by comparing the comet images with similarly exposed plates of the Polar Sequence.

Comet Harrington (1935 II, 1957 *g*) was not found on Crossley plates taken March 13, 1959. It must then have been fainter than magnitude 18.

							Par. factor		
UT	α			δ			α	δ	
	Comet Schwassmann-Wachmann I (1925 II)								
1958–1959	1950.0								
Aug. 21.35362	23^h	47^m	$54^s.41$	$+5°$	17′	$00''.3$	$-0^s.178$	$+4''.69$	J,G
21.39668	23	47	53.45	+5	16	59.2	−0.055	+4.64	F,G
Nov. 4.24302	23	20	11.09	+2	59	13.3	+0.150	+4.95	J,G
4.28677	23	20	10.69	+2	59	09.3	+0.265	+5.00	J,G
Aug. 7.43925	1	37	00.16	+19	18	10.8	−0.267	+3.06	J,G
7.48233	1	37	00.32	+19	18	16.8	−0.146	+2.79	J,G
Oct. 27.22122	1	13	18.86	+18	47	42.5	−0.214	+2.99	G,G
27.30039	1	13	16.82	+18	47	30.6	+0.025	+2.76	G,G
	Comet Oterma (1942 VII)								
1958–1959	1950.0								
Apr. 9.29161	8^h	19^m	$29^s.49$	$+18°$	20′	$23''.5$	$+0^s.405$	$+3''.79$	J,J
9.31800	8	19	30.13	+18	20	23.2	+0.445	+4.11	J,J
May 8.20170	8	38	03.04	+17	44	02.3	+0.358	+3.60	J,F
8.24059	8	38	05.00	+17	43	56.9	+0.428	+4.02	J,F
Mar. 13.45134	13	33	54.57	−6	06	06.1	+0.062	+6.02	J,G
13.48398	13	33	53.71	−6	05	58.7	+0.156	+5.98	J,G
May 29.23281	12	58	10.62	−1	58	24.4	+0.109	+5.55	J,G
29.28491	12	58	10.33	−1	58	24.3	+0.250	+5.51	G,G
	Comet Wirtanen (1956 *c*)								
1958–1959	1950.0								
April 26.47064	18^h	59^m	$54^s.88$	$+2°$	58′	$54''.1$	$-0^s.185$	$+4''.97$	J,J
26.49216	18	59	55.02	+2	59	04.4	−0.126	+4.95	J,J
June 21.39559	18	49	34.10	+9	08	29.5	+0.061	+4.14	J,F
21.42198	18	49	33.38	+9	08	35.6	+0.137	+4.18	F,F
21.44559	18	49	32.81	+9	08	40.9	+0.203	+4.24	J,F
Aug. 12.21954	18	31	12.07	+9	41	25.3	−0.003	+4.05	J,G
12.30220	18	31	11.16	+9	41	14.4	+0.233	+4.21	F,G
Sept. 5.20854	18	30	25.44	+8	37	20.8	+0.158	+4.26	F,G
5.25646	18	30	25.69	+8	37	11.8	+0.283	+4.41	F,G
Nov. 4.13538	18	55	31.44	+6	00	21.4	+0.337	+4.77	J,G
4.17983	18	55	33.52	+6	00	17.3	+0.415	+4.94	J,G
June 6.39722	21	04	45.83	+20	06	32.5	−0.325	+3.16	J,J
6.44653	21	04	45.62	+20	06	46.9	−0.195	+2.76	J,J
July 3.38907[a]	20	59	48.49	+21	44	41.1	−0.138	+2.43	J,G
3.43560	20	59	49.62	+21	44	59.6	+0.007	+2.33	G,G
July 9.38777	20	58	02.03	+21	57	28.5	−0.088	+2.34	J,J
9.43777	20	58	01.15	+21	57	32.8	+0.070	+2.33	J,J
	Comet Mrkos (1957 *d*)								
1958	1950.0								
April 26.41575	16^h	55^m	$32^s.53$	$-31°$	00′	$10''.1$	$-0^s.114$	$+8''.08$	J,G
26.43590	16	55	31.02	−31	00	14.4	−0.046	+8.15	J,G

---

* *Lick Observatory Bulletin*, No. 565.

				Comet Reinmuth I (1957 *e*)					
1958				1950.0					
April 9.21936	7^h	40^m	$11^s.17$	+24°	43′	$10''.7$	$+0^s.324$	$+2''.55$	J,G
April 18.23902	7	56	08.19	+24	28	27.1	+0.400	+3.01	J,G
18.27792	7	56	12.43	+24	28	22.5	+0.465	+3.61	J,G
				Comet Burnham (1958 *a*)					
1958				1950.0					
April 18.18346	8^h	09^m	$06^m.50$	+27°	56′	$41''.8$	$+0^s.246$	$+1''.79$	J,G
18.19804	8	09	09.69	+27	56	50.1	+0.288	+1.94	J,G
				Comet Kopff (1958 *d*)					
1958				1950.0					
Aug. 21.45572	2^h	42^m	$42^s.66$	+9°	48′	$38''.8$	$-0^s.231$	$+4''.18$	J,G
21.49601	2	42	43.37	+9	48	36.8	−0.119	+4.07	J,G
Sept. 16.46956	2	41	12.77	+8	43	02.6	+0.015	+4.19	J,G
16.51054	2	41	11.66	+8	42	53.1	+0.136	+4.23	J,G
				Comet Slaughter-Burnham (1959 *a*)					
1959				1950.0					
Mar. 3.19836	3^h	42^m	$24^s.25$	+27°	57′	$12''.7$	$+0^s.429$	$+2''.77$	J,G
3.25740	3	42	29.3	+27	57	18	+0.512	+3.87	J,G
				Comet Giacobini-Zinner (1959 *b*)					
1959				1950.0					
June 12.39598	17^h	27^m	$37^s.80$	+28°	10′	$16''.2$	$+0^s.171$	$+1''.55$	J,J
12.44459	17	27	33.91	+28	11	08.0	+0.315	+2.02	G,J
				Comet Alcock (1959 *e*)					
1959				1950.0					
Sept. 1.19153	16^h	25^m	$46^s.08$	+28°	15′	$00''.7$	$+0^s.343$	$+2''.15$	G,G
1.21549	16	25	53.36	+28	13	25.0	+0.400	+2.51	G,G
				Comet Schaumasse (1959 *h*)					
1959				1950.0					
Sept. 30.43715	3^h	12^m	$28^s.24$	+2°	43′	$03''.0$	$-0^s.032$	$+4''.97$	J,G
30.50208	3	12	27.24	+2	42	45.3	+0.155	+4.99	G,G
Oct. 8.31846	3	09	49.94	+2	07	56.0	−0.290	+5.10	G,J
8.44844	3	09	46.32	+2	07	18.6	+0.070	+5.04	G,J
27.34763	2	55	52.36	+0	37	06.0	−0.046	+5.23	G,J
27.40804	2	55	48.54	+0	36	48.9	+0.130	+5.23	G,J
				Icarus (1566)					
1958				1950.0					
June 25.40766	18^h	46^m	$50^s.46$	−33°	45′	$31''.3$	$+0^s.158$	$+8''.15$	F,F
25.42572	18	46	42.45	−33	45	54.3	+0.218	+8.00	J,F
25.44516	18	46	34.06	−33	46	18.5	+0.280	+7.79	J,F

[a] The measure of July 3.38907 refers to the companion nucleus. The primary nucleus was unmeasurable due to coincidence with a star trail. Al other positions given above are of the primary or brighter nucleus.

NOTES

Comet Schwassmann-Wachmann I (1925 II)

Aug. 21: Stellar nucleus, with fan-shaped coma extending $0'.5$ East. Magnitude of nucleus was 17.5, total magnitude was about 15.

Sept. 16: A round coma about 1′ in diameter. Magnitude of nucleus was about 17.5.

Nov. 4: Stellar nucleus of 17.5 mag with faint coma nearly 1′ in diameter.

Aug. 7: Nearly stellar, magnitude 19.

Oct. 23: A 15-mag nucleus, with an irregular coma nearly 4′ across.

Oct. 27: Total magnitude about 10, otherwise as on October 23. Visible in the 4-in. finder.

Nov. 3: Stellar nucleus of 17.8 mag, with coma about 3′ across. Coma not so bright as on October 23 and 27. Total magnitude was about 14.

Nov. 23: Appearance similar to that on Nov. 3, except that it was fainter and smaller. Total magnitude was about 16.

Comet Oterma (1942 VII)

April 9, May 8: Nearly stellar, magnitude 17.5.

March 13: Magnitude 17, with a faint tail extending $0'.6$ in 300°.

May 29: Appearance as on April 9 and May 8.

Comet Wirtanen (1956 *c*)

April 26: Nuclei of 16 and 17.3 mag, not quite stellar. A shaped coma extends $0'.5$ northeast of the primary nuc

June 21: Appearance as on April 26.

Aug. 12: Both components had sharp nuclei, with a small c surrounding each. Magnitudes 16.3 and 17.7.

Sept. 5: Magnitudes 17 and 18.

Nov. 4: Magnitudes 17.5 and 18.5, and there is still a f coma northeast of the brighter component.

June 6: Components of magnitude 18 and 19, both slig diffuse.

July 9: Appearance as on June 6.

The following are the measured differences in right ascen and declimation between the two components. The faint second component was always west and south of the prim

1958 April 26	$\Delta\alpha = 1^s.02$	$\Delta\delta = 13''.0$
June 21	1.22	14.7
Aug. 12	1.17	14.6
Sept. 5	1.10	13.9
Nov. 4	1.08	11.7
1959 June 6	1.79	9.5
July 9	1.88	10.4

Comet Mrkos (1957 *d*)

April 26: The comet shows a round coma $0'.2$ in diameter, a sharp nucleus of magnitude 17.5.

Comet Reinmuth (1957 *e*)

April 9, 18: The comet was slightly diffuse, of magnitude 1

met Burnham (1958 *a*)

pril 18: A 12-minute exposure shows a round coma, 2′ across, with a well-condensed center. The total magnitude was about 12.

met Kopff (1958 *d*)

ug. 21: Shows a nearly stellar nucleus of magnitude 19, with a very faint coma 0′.2 across.

met Slaughter-Burnham (1959 *a*)

larch 3: Comet diffuse, of magnitude 19.

Comet Giacobini-Zinner (1959 *b*)

June 12: The comet shows a nearly stellar image of magnitude 18.

Comet Alcock (1959 *e*)

Sept. 1: Round, 3′ in diameter, centrally condensed, and of total magnitude about 10.5.

Comet Schaumasse (1959 *h*)

Sept. 30: Nearly stellar, about 19.4 mag.
Oct. 27: Nearly stellar, about 18.5 mag.

E ASTRONOMICAL JOURNAL VOLUME 65, NUMBER 3 APRIL, 1960

Observations of Comet Väisälä, 1939 IV, 1959 i

JAMES GIBSON
Lick Observatory, Mount Hamilton, California

The comet was recovered November 11, 1959 at Lick Observatory. Subsequent observations through March 1960 are presented.

THIS paper is a companion to the recent paper containing observations of comets, Lick Observatory Bulletin No. 565, published in this issue of the Astronomical Journal. The observations which are presented in Table I were also made with the Crossley reflector.

The comet observed is not the object announced by another observatory on November 4, 1959 as being P/Väisälä (H.A.C. No. 1454, U.A.I.C. No. 1697). For details of the correct identification see H.A.C. Nos. 1455 and 1458, U.A.I.C. Nos. 1698 and 1702. Observations from the U. S. Naval Observatory, Flagstaff, have also been reported (H.A.C. No. 1462, U.A.I.C. No. 1705).

TABLE I.

UT 1959–60	α 1950.0	δ 1950.0	Par. factor α	Par. factor δ
Nov. 11.50662	$8^{\mathrm{h}}\ 29^{\mathrm{m}}\ 03^{\mathrm{s}}.77$	$+11^\circ\ 40'\ 48''.5$	$-0^{\mathrm{s}}.138$	$+3''.84$
12.53696	8 30 05.84	+11 37 35.0	−0.042	+3.80
Dec. 1.54387	8 46 29.82	+10 52 25.5	+0.099	+3.93
3.49833	8 47 50.52	+10 49 56.6	−0.023	+3.90
3.55319	8 47 52.49	+10 49 53.2	+0.139	+3.96
Jan. 5.44993	8 57 47.41	+11 53 30.5	+0.081	+3.78
5.49022	8 57 47.06	+11 53 44.0	+0.197	+3.88
Feb. 16.23801	8 38 52.10	+19 00 50.5	−0.174	+2.89
16.25607	8 38 51.44	+19 01 04.3	−0.120	+2.81
Mar. 20.16177	8 39 26.97	+24 18 46.0	−0.139	+2.07
20.17913	8 39 27.61	+24 18 52.1	−0.084	+2.00

Notes

Nov. 11, 12: Stellar image, magnitude 19.5.

Dec. 1, 3: Stellar image, magnitude 19.0.

Jan. 5: Not quite stellar, with a faint tail 0′.4 long in 290°. Magnitude 17.5.

Feb. 16: Exposure 20 minutes, plate fogged by moonlight. A faint coma, but no tail visible.

March 20: Condensed, but with a very faint coma or tail in 100°. Exposure 20 minutes. Magnitude 17.